

A list of close to AONT matrices found by computer search

Navid Nasr Esfahani and Douglas R. Stinson*

David R. Cheriton School of Computer Science
University of Waterloo
Waterloo, Ontario N2L 3G1, Canada

October 14, 2016

Abstract

This is the supplementary document for our study of All-or-Nothing transforms presented in [1, 2]. The most up-to-date list of our computational constructions will be presented in this document.

*D. Stinson's research is supported by an NSERC discovery grant.

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1 $q = 2$

1.1 Random Construction

An invertible 3 by 3 matrix having 7 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

An invertible 4 by 4 matrix having 30 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 5 by 5 matrix having 70 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

An invertible 6 by 6 matrix having 147 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

An invertible 7 by 7 matrix having 274 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

An invertible 8 by 8 matrix having 473 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 4 by 4 matrix with 30 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

An invertible 5 by 5 matrix with 65 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

An invertible 6 by 6 matrix with 135 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 7 by 7 matrix with 287 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 8 by 8 matrix with 468 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 9 by 9 matrix with 765 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 31 by 31 matrix with 113863 invertible 2×2 submatrices:

0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	1	1	1	1	1	1	0	1	0	1	1	1																
0	0	0	1	1	0	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	0	1	0	1	1	1	0															
0	0	1	1	0	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	0	1	0	1	1	1	0	0															
0	1	1	0	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	0	1	0	1	1	1	1	0	0	0														
1	1	0	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	0	1	0	1	1	1	1	0	0	0	0														
1	0	1	1	1	1	0	1	1	0	1	1	1	0	1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	1													
0	1	1	1	1	0	1	1	0	1	1	1	0	1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	1	1													
1	1	1	1	0	1	1	0	1	1	1	0	1	1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	1	1	0												
1	1	1	0	1	1	0	1	1	1	0	1	1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	1	1	0	1												
1	1	0	1	1	0	1	1	1	0	1	1	1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	1	1	0	1	1											
1	0	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	1	1	0	1	1	1										
0	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	1	1	0	1	1	1	1									
1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	1	1	0	1	1	1	1	0									
1	0	1	1	1	0	1	1	1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	1							
0	1	1	1	0	1	1	1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	0	1	1	0	0	0	1	1	1	1	0	1	1	1	1						
1	1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	1	1	0	1	1	1	0	1	1	0					
1	1	1	1	1	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	1	1	0	1	1	1	0	1	1	1	0	1				
1	1	1	1	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	1	1	0	1	1	0	1	1	0	1	1	1	0	1	1	1	0	1	1	1				
1	1	1	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	1	1	0	1	1	0	1	1	0	1	1	1	0	1	1	1	0	1	1	1	1				
1	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1				
0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1				
1	0	1	1	1	0	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	
0	1	1	1	0	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	1	0	
1	1	1	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	
1	1	0	0	0	0	1	1	0	1	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	
1	0	0	0	0	1	1	0	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1

An invertible 5 by 5 matrix with 69 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 6 by 6 matrix with 148 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

An invertible 7 by 7 matrix with 287 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 8 by 8 matrix with 485 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 9 by 9 matrix with 781 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 10 by 10 matrix with 1215 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 8 by 8 matrix with 485 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

An invertible 9 by 9 matrix with 783 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

An invertible 10 by 10 matrix with 1194 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

2 $q = 3$

2.1 Random Construction

An invertible 3 by 3 matrix with 9 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$$

An invertible 4 by 4 matrix with 34 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 2 & 2 & 2 \\ 1 & 0 & 2 & 2 \\ 2 & 1 & 2 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix}$$

An invertible 5 by 5 matrix with 86 invertible 2×2 submatrices:

$$\begin{pmatrix} 2 & 1 & 2 & 2 & 0 \\ 1 & 0 & 2 & 1 & 2 \\ 1 & 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 1 & 2 \end{pmatrix}$$

An invertible 6 by 6 matrix with 185 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 0 & 1 & 2 & 1 & 1 \\ 2 & 2 & 1 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 & 0 & 2 \\ 2 & 1 & 0 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 & 2 \\ 2 & 1 & 2 & 2 & 2 & 0 \end{pmatrix}$$

An invertible 7 by 7 matrix with 343 invertible 2×2 submatrices:

$$\begin{pmatrix} 2 & 2 & 1 & 0 & 0 & 1 & 2 \\ 1 & 1 & 2 & 1 & 0 & 2 & 2 \\ 1 & 2 & 0 & 2 & 2 & 0 & 2 \\ 2 & 0 & 1 & 1 & 2 & 2 & 2 \\ 1 & 2 & 1 & 1 & 2 & 2 & 0 \\ 0 & 1 & 1 & 1 & 2 & 1 & 2 \\ 1 & 1 & 0 & 0 & 2 & 1 & 1 \end{pmatrix}$$

An invertible 8 by 8 matrix with 591 invertible 2×2 submatrices:

$$\begin{pmatrix} 2 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 1 & 0 & 0 & 1 & 1 \\ 2 & 1 & 1 & 1 & 2 & 0 & 2 & 1 \\ 2 & 2 & 1 & 1 & 1 & 2 & 1 & 0 \\ 2 & 2 & 2 & 2 & 0 & 1 & 2 & 2 \\ 2 & 0 & 2 & 2 & 1 & 2 & 2 & 0 \\ 0 & 1 & 2 & 2 & 1 & 2 & 1 & 1 \\ 2 & 1 & 0 & 2 & 2 & 2 & 1 & 2 \end{pmatrix}$$

An invertible 9 by 9 matrix with 965 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 2 & 2 & 0 & 1 & 0 & 1 \\ 2 & 2 & 2 & 2 & 0 & 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 2 & 2 & 1 & 2 & 2 & 1 \\ 0 & 2 & 1 & 1 & 2 & 1 & 0 & 2 & 2 \\ 1 & 0 & 2 & 1 & 2 & 0 & 1 & 2 & 1 \\ 2 & 2 & 0 & 0 & 2 & 2 & 1 & 2 & 2 \\ 2 & 2 & 1 & 1 & 1 & 2 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 & 2 & 2 & 2 & 2 & 0 \end{pmatrix}$$

An invertible 10 by 10 matrix with 1479 invertible 2×2 submatrices:

$$\begin{pmatrix} 2 & 1 & 2 & 1 & 2 & 0 & 2 & 1 & 0 & 1 \\ 2 & 2 & 0 & 1 & 2 & 2 & 1 & 1 & 1 & 2 \\ 2 & 2 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 1 & 2 & 2 & 1 & 1 & 2 & 0 \\ 1 & 2 & 2 & 2 & 2 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 & 2 & 2 & 2 & 1 & 0 & 1 \\ 2 & 2 & 2 & 1 & 1 & 0 & 2 & 2 & 2 & 0 \\ 2 & 2 & 2 & 0 & 2 & 1 & 0 & 1 & 2 & 2 \end{pmatrix}$$

An invertible 11 by 11 matrix with 2189 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 2 & 2 \\ 2 & 1 & 2 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 2 \\ 0 & 2 & 2 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 0 \\ 2 & 0 & 0 & 1 & 1 & 1 & 2 & 1 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 & 0 & 1 & 2 & 2 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 2 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 & 2 & 1 & 0 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 2 & 2 & 2 & 2 & 0 & 0 & 1 \\ 1 & 2 & 2 & 0 & 1 & 0 & 0 & 2 & 2 & 1 & 1 \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 2 & 2 & 1 & 1 \\ 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 & 2 & 2 & 2 \end{pmatrix}$$

An invertible 12 by 12 matrix with 3090 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 2 & 1 & 2 & 2 & 2 & 1 & 2 & 1 \\ 2 & 0 & 2 & 1 & 0 & 0 & 2 & 1 & 1 & 2 & 2 & 1 \\ 0 & 2 & 1 & 2 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 2 & 2 & 2 & 0 & 1 & 0 & 1 & 1 \\ 2 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 & 2 & 2 & 0 & 2 \\ 2 & 0 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 2 & 2 & 0 & 2 & 1 & 2 & 1 & 2 & 2 & 1 \\ 0 & 2 & 0 & 2 & 2 & 1 & 1 & 2 & 2 & 1 & 1 & 0 \\ 2 & 2 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 2 \\ 2 & 2 & 2 & 1 & 0 & 0 & 2 & 0 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 2 & 2 & 0 & 1 & 2 & 2 & 2 & 2 & 1 \end{pmatrix}$$

An invertible 13 by 13 matrix with 4306 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 1 & 0 & 2 & 2 & 1 & 2 & 2 & 2 & 2 & 0 & 1 & 2 \\ 2 & 1 & 2 & 0 & 1 & 2 & 2 & 2 & 1 & 1 & 0 & 2 & 1 \\ 2 & 2 & 1 & 0 & 2 & 1 & 0 & 1 & 2 & 0 & 2 & 1 & 2 \\ 0 & 1 & 2 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 0 \\ 1 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 0 & 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 0 & 1 & 2 & 1 & 2 & 2 & 2 & 1 & 1 & 2 & 0 & 1 & 2 \\ 2 & 1 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 0 & 2 & 1 & 2 \\ 1 & 0 & 2 & 2 & 1 & 1 & 0 & 2 & 1 & 2 & 2 & 1 & 1 \\ 1 & 1 & 2 & 0 & 2 & 1 & 1 & 2 & 2 & 2 & 1 & 2 & 0 \\ 1 & 1 & 0 & 1 & 2 & 2 & 2 & 0 & 0 & 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 & 2 & 0 & 2 & 1 & 2 & 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & 2 & 1 & 1 & 0 & 0 & 1 & 2 & 2 & 0 & 1 \end{pmatrix}$$

2.2 Cyclic Matrices

An invertible 3 by 3 matrix with 9 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

An invertible 4 by 4 matrix with 34 invertible 2×2 submatrices:

$$\begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 1 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix}$$

An invertible 5 by 5 matrix with 90 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 & 0 \\ 1 & 1 & 2 & 0 & 1 \\ 1 & 2 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 6 by 6 matrix with 189 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 2 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 & 1 & 0 \\ 2 & 2 & 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & 2 & 2 & 1 \end{pmatrix}$$

An invertible 7 by 7 matrix with 357 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 2 & 1 & 2 & 0 \\ 1 & 1 & 2 & 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 & 1 \\ 2 & 1 & 2 & 0 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 & 1 & 2 & 1 \end{pmatrix}$$

An invertible 8 by 8 matrix with 600 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 2 & 2 \\ 1 & 0 & 1 & 1 & 1 & 2 & 2 & 0 \\ 0 & 1 & 1 & 1 & 2 & 2 & 0 & 1 \\ 1 & 1 & 1 & 2 & 2 & 0 & 1 & 0 \\ 1 & 1 & 2 & 2 & 0 & 1 & 0 & 1 \\ 1 & 2 & 2 & 0 & 1 & 0 & 1 & 1 \\ 2 & 2 & 0 & 1 & 0 & 1 & 1 & 1 \\ 2 & 0 & 1 & 0 & 1 & 1 & 1 & 2 \end{pmatrix}$$

An invertible 9 by 9 matrix with 1008 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 2 & 2 & 2 & 1 & 1 & 2 & 1 \\ 0 & 2 & 2 & 2 & 1 & 1 & 2 & 1 & 0 \\ 2 & 2 & 2 & 1 & 1 & 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 1 & 2 & 1 & 0 & 0 & 2 \\ 2 & 1 & 1 & 2 & 1 & 0 & 0 & 2 & 2 \\ 1 & 1 & 2 & 1 & 0 & 0 & 2 & 2 & 2 \\ 1 & 2 & 1 & 0 & 0 & 2 & 2 & 2 & 1 \\ 2 & 1 & 0 & 0 & 2 & 2 & 2 & 1 & 1 \\ 1 & 0 & 0 & 2 & 2 & 2 & 1 & 1 & 2 \end{pmatrix}$$

An invertible 10 by 10 matrix with 1550 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 2 & 2 & 2 & 1 & 1 & 2 & 1 & 2 \\ 0 & 2 & 2 & 2 & 1 & 1 & 2 & 1 & 2 & 0 \\ 2 & 2 & 2 & 1 & 1 & 2 & 1 & 2 & 0 & 0 \\ 2 & 2 & 1 & 1 & 2 & 1 & 2 & 0 & 0 & 2 \\ 2 & 1 & 1 & 2 & 1 & 2 & 0 & 0 & 2 & 2 \\ 1 & 1 & 2 & 1 & 2 & 0 & 0 & 2 & 2 & 2 \\ 1 & 2 & 1 & 2 & 0 & 0 & 2 & 2 & 2 & 1 \\ 2 & 1 & 2 & 0 & 0 & 2 & 2 & 2 & 1 & 1 \\ 1 & 2 & 0 & 0 & 2 & 2 & 2 & 1 & 1 & 2 \\ 2 & 0 & 0 & 2 & 2 & 2 & 1 & 1 & 2 & 1 \end{pmatrix}$$

An invertible 5 by 5 matrix with 90 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 & 0 \\ 1 & 1 & 2 & 0 & 1 \\ 1 & 2 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 6 by 6 matrix with 189 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 1 & 2 & 0 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 0 & 1 & 1 \\ 1 & 2 & 0 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

An invertible 7 by 7 matrix with 357 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 2 & 1 & 2 & 0 \\ 1 & 1 & 2 & 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 & 1 \\ 2 & 1 & 2 & 0 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 & 1 & 2 & 1 \end{pmatrix}$$

An invertible 8 by 8 matrix with 608 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 2 & 2 & 1 \\ 0 & 1 & 1 & 1 & 2 & 2 & 1 & 0 \\ 1 & 1 & 1 & 2 & 2 & 1 & 0 & 0 \\ 1 & 1 & 2 & 2 & 1 & 0 & 0 & 1 \\ 1 & 2 & 2 & 1 & 0 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 & 0 & 1 & 1 & 1 \\ 2 & 1 & 0 & 0 & 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 & 1 & 1 & 2 & 2 \end{pmatrix}$$

An invertible 9 by 9 matrix with 1008 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 2 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 & 2 & 2 & 1 & 2 & 0 \\ 1 & 1 & 1 & 2 & 2 & 1 & 2 & 0 & 0 \\ 1 & 1 & 2 & 2 & 1 & 2 & 0 & 0 & 1 \\ 1 & 2 & 2 & 1 & 2 & 0 & 0 & 1 & 1 \\ 2 & 2 & 1 & 2 & 0 & 0 & 1 & 1 & 1 \\ 2 & 1 & 2 & 0 & 0 & 1 & 1 & 1 & 2 \\ 1 & 2 & 0 & 0 & 1 & 1 & 1 & 2 & 2 \\ 2 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 1 \end{pmatrix}$$

An invertible 10 by 10 matrix with 1550 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 2 & 2 & 1 & 2 & 0 & 0 \\ 1 & 1 & 1 & 2 & 2 & 1 & 2 & 0 & 0 & 1 \\ 1 & 1 & 2 & 2 & 1 & 2 & 0 & 0 & 1 & 1 \\ 1 & 2 & 2 & 1 & 2 & 0 & 0 & 1 & 1 & 1 \\ 2 & 2 & 1 & 2 & 0 & 0 & 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 0 & 0 & 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 0 & 0 & 1 & 1 & 1 & 1 & 2 & 2 \\ 2 & 0 & 0 & 1 & 1 & 1 & 1 & 2 & 2 & 1 \end{pmatrix}$$

An invertible 5 by 5 matrix with 90 invertible 2×2 submatrices:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 2 & 1 \\ 1 & 1 & 2 & 0 & 2 \\ 1 & 2 & 2 & 1 & 0 \end{pmatrix}$$

References

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