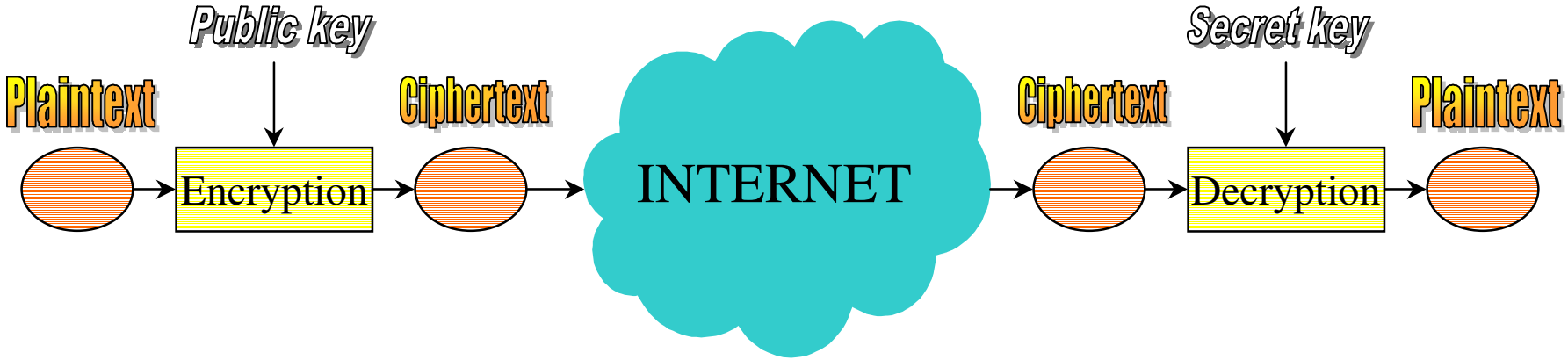
DHIES: An Encryption Scheme based on the Diffie-Hellman problem

Michel Abdalla (UC San Diego)
Mihir Bellare (UC San Diego)
Phillip Rogaway (UC Davis)

Public-key Encryption



Diffie-Hellman/Discrete Log based encryption

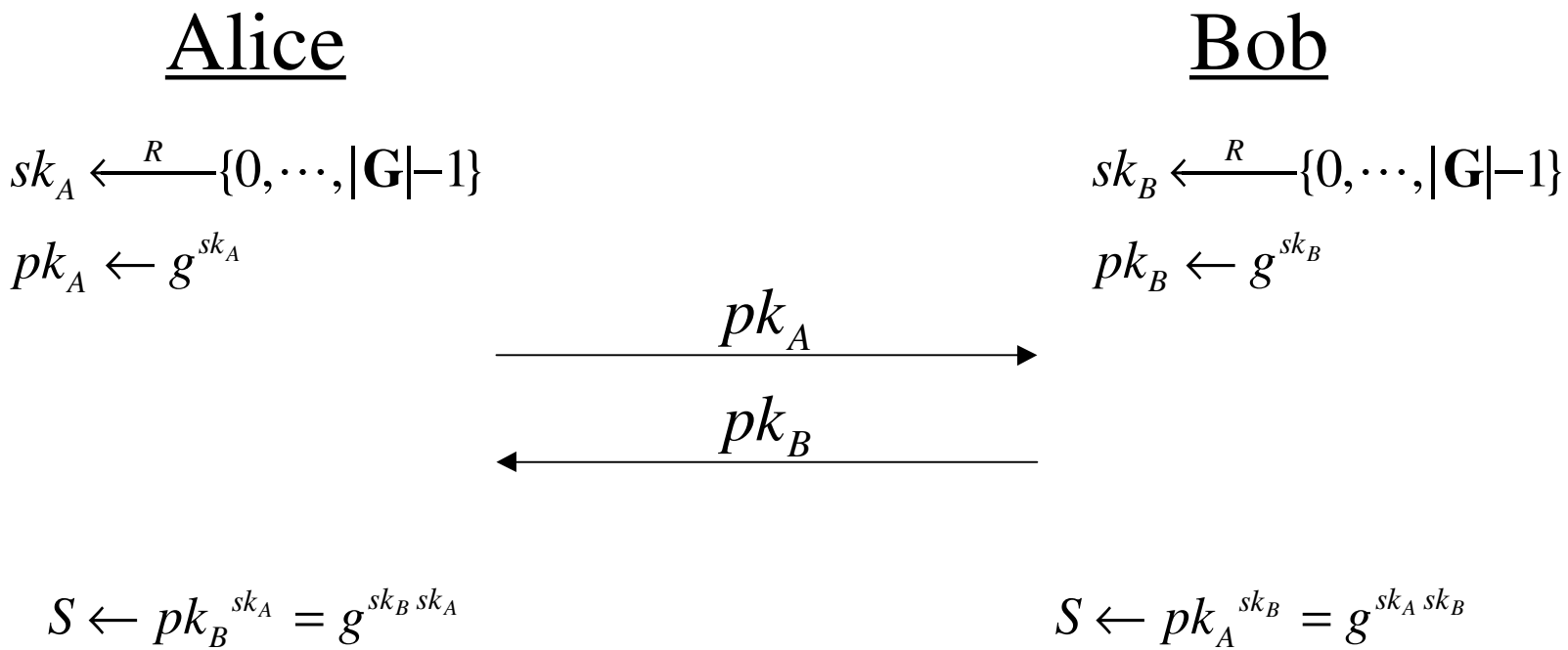
Why?

- Low cost: elliptic curve groups of relatively small size suffice.

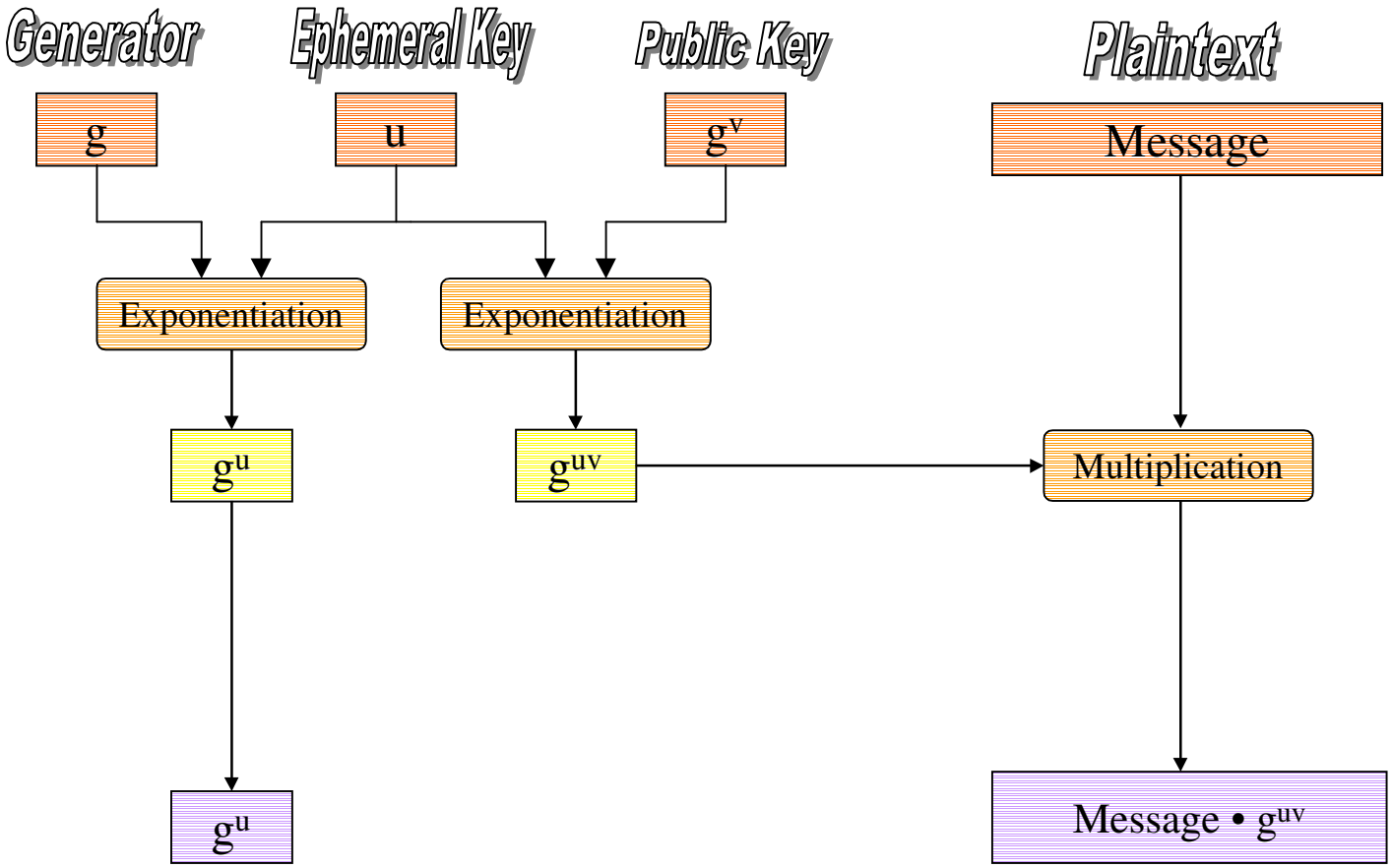
- Alternative to RSA.

Diffie-Hellman Key Exchange Protocol

Let g be a generator for a group $\mathrm{G} = \{g^0, g^1, \cdots, g^{|\mathrm{G}|-1}\}$



El Gamal Encryption Scheme



Deficiencies of the El Gamal Scheme

- Security weakness: susceptible to chosen-ciphertext attacks.
- Functional weakness: requires message to be a group element.

Our Goal

Design a natural, simple extension of the El Gamal scheme.

- Security attributes:
  - resistance to chosen-ciphertext attacks.
  - security supported by a proof.
- Functional attributes:
  - Can encrypt arbitrary messages
  - Same cost as El Gamal (for public key operations).

Components of DHIES

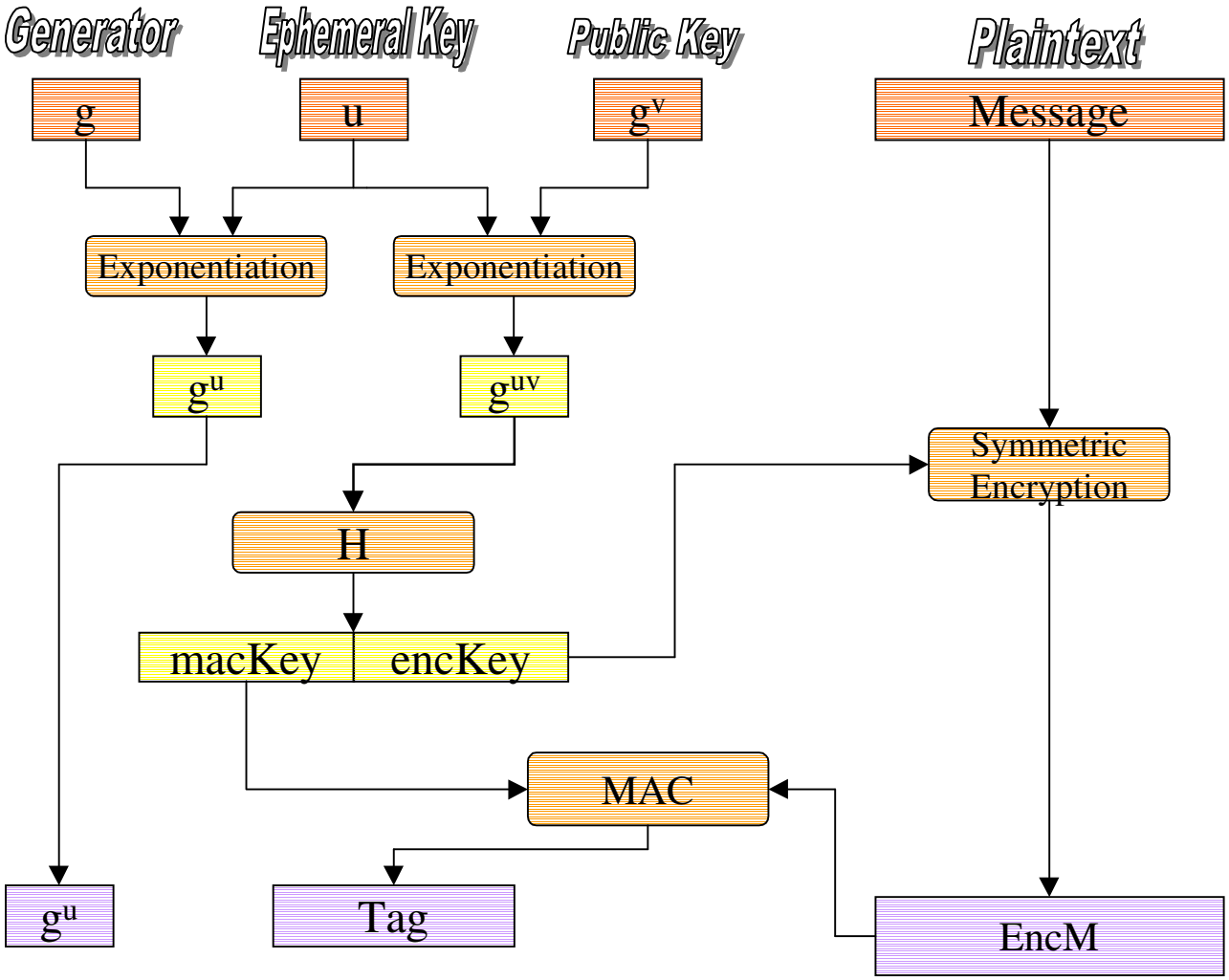
- *A Symmetric Encryption Scheme*.

  e.g. DES/CBC encryption.
- *A Message Authentication Scheme*.

  e.g. HMAC.
- *H (Secure Hash Function)*.

  e.g. SHA-1/MD5.

Our scheme: DHIES



Intuition

- MAC prevents chosen-ciphertext attacks.
- Symmetric encryption enables encryption of arbitrary messages.

Use of DHIES

- ANSI X9.63EC (draft) standard
- IEEE P1363a (draft) standard
- Corporate (draft) standard SEC

Comparison of Schemes in Groups of Prime Order

Scheme	IND-CPA	IND-CCA	Underlying Assumption	Cost: Number of Mod. Exp's
El Gamal	✓	X	DDH	2
Cramer-Shoup	✓	✓	DDH	4
DHIES	✓	✓	ODH	2

Rest of the talk

- Measures of security for public-key encryption schemes
- Assumptions about hardness of discrete log/Diffie-Hellman related problems
- Security of DHIES

Security Goals

- Indistinguishability (IND)

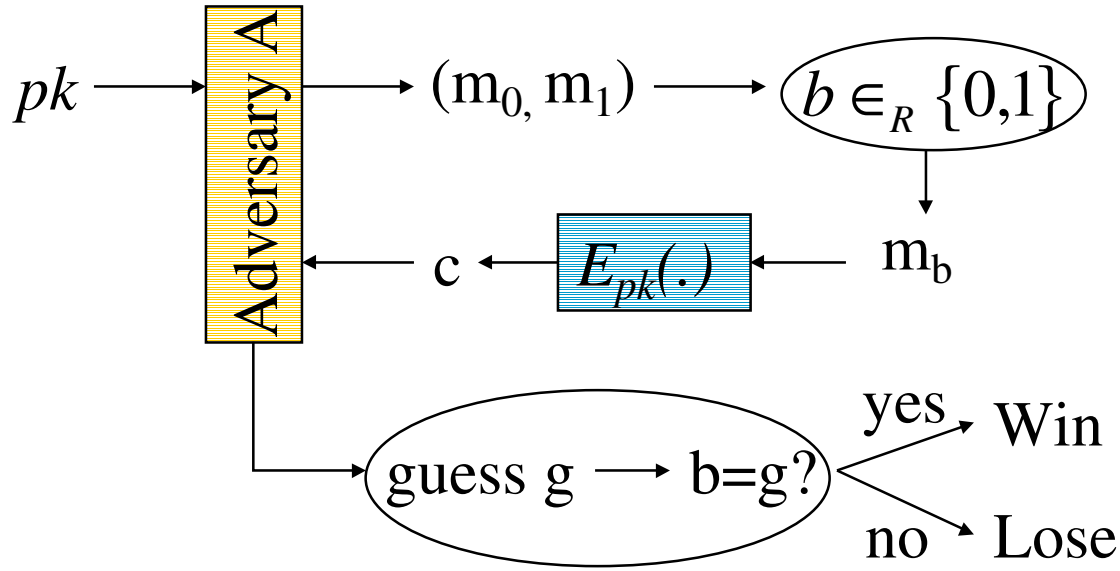
  The encryption of two different plaintexts of the same length should be indistinguishable.

- Types of attack
  - Chosen-plaintext attack (CPA)

    Always possible with PK encryption
  - Chosen-ciphertext attack (CCA)

    Adversary can make queries to a decryption oracle

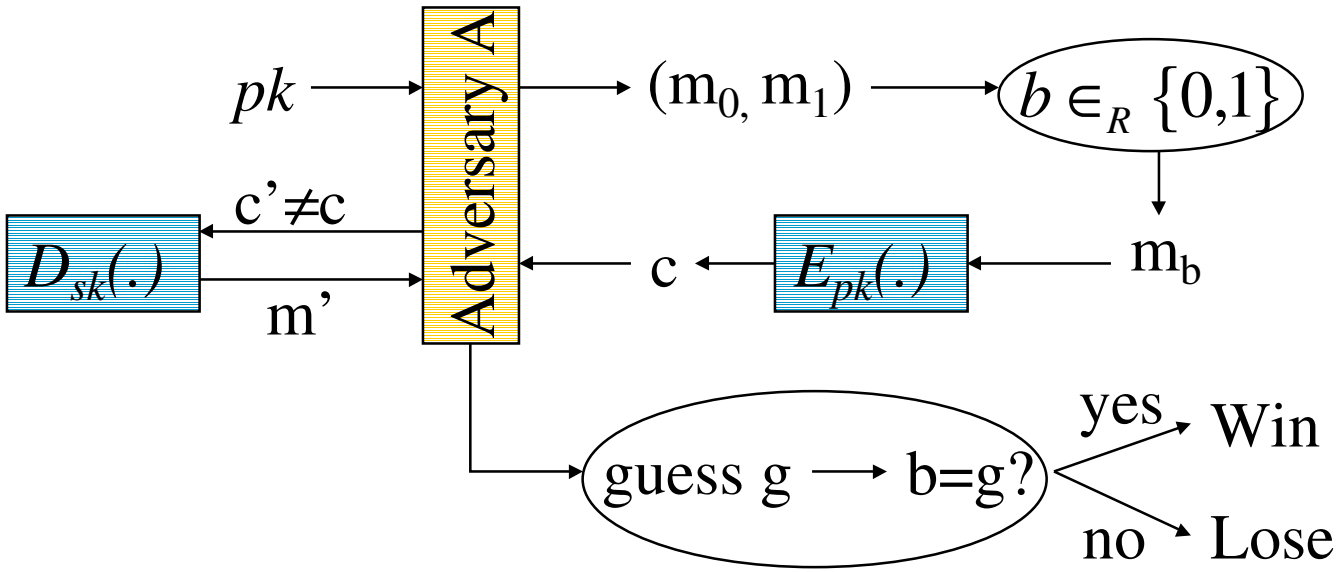
IND-CPA



$$\mathrm{Adv}^{\mathrm{ind-cpa}}_{SCHEME}(t) = \max_{A \text{ running in time } t} \{\Pr[A \text{ wins}]\}$$

Scheme is IND-CPA secure if $\mathrm{Adv}^{\mathrm{ind-cpa}}_{SCHEME}(t)$ is "small" for "large" *t.*

IND-CCA



$$\mathrm{Adv}^{\mathrm{ind-cca}}_{SCHEME}(t,q) = \max_{\substack{A \text{ running in time } t \\ \text{and making } q \text{ queries}}} \{\Pr[A \text{ wins}]\}$$

Scheme is IND-CCA secure if $\mathrm{Adv}^{\mathrm{ind-cca}}_{SCHEME}(t)$ is "small" for "large" *t.*

Summary

- Measures of security associated to *SCHEME*,

$$\mathrm{Adv}^{\mathrm{ind-cpa}}_{SCHEME}(t) \quad \text{and} \quad \mathrm{Adv}^{\mathrm{ind-cca}}_{SCHEME}(t,q),$$

  capture strong security requirements (much beyond hardness of recovery of secret key) in a quantitative way.
- Practice-oriented provable-security approach.

Rest of the Talk

✓ Measures of security for public-key encryption schemes

- Assumptions about hardness of discrete log/Diffie-Hellman related problems

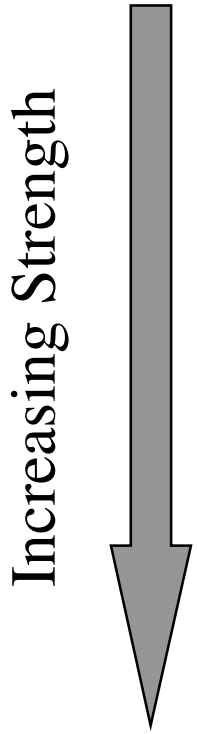
- Security of DHIES

- We seek clear, believable assumptions on the DH problem under which security of natural, simple schemes (e.g., DHIES) can be proved.

- Our assumptions are new and stronger than previous ones, but this has enabled low-cost schemes with proven security.

- We do not use random oracles!

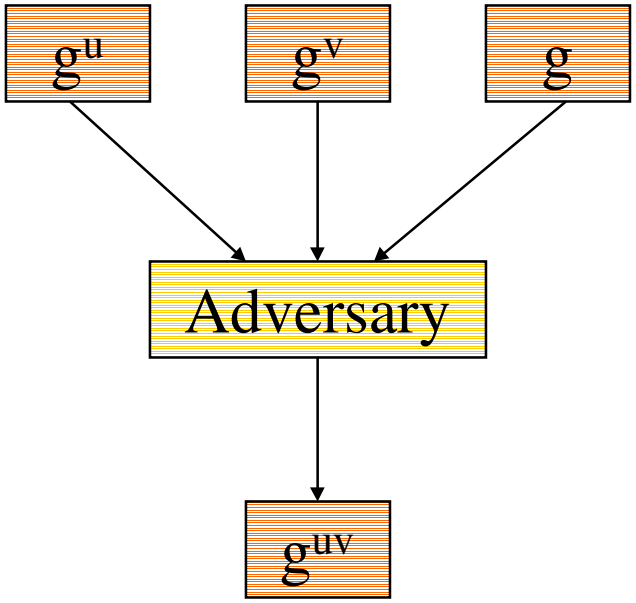
Assumptions



- CDH - Computational Diffie-Hellman
- DDH - Deciosnal Diffie-Hellman
- HDH - Hash Diffie-Hellman
- ODH - Oracle Diffie-Hellman

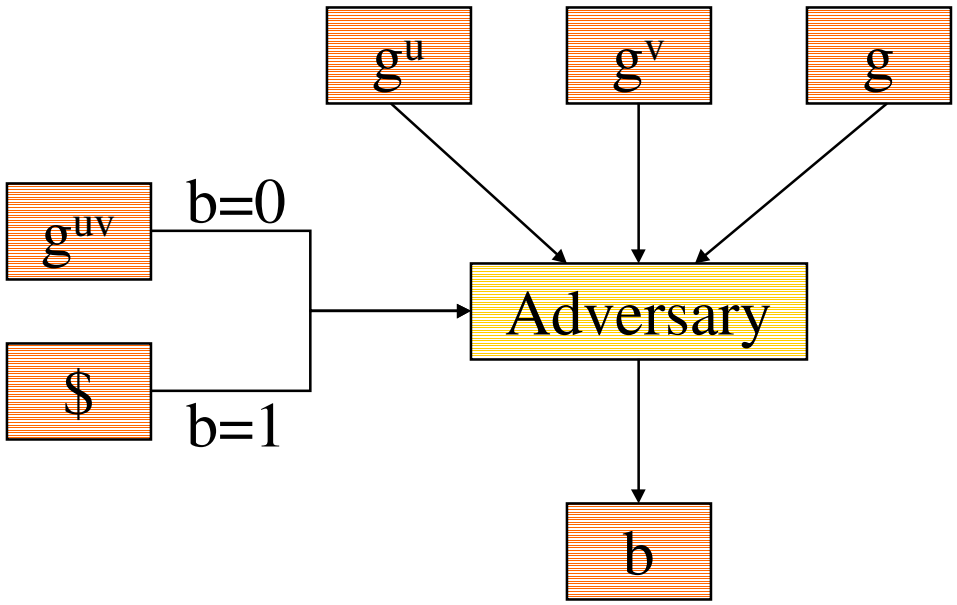
Computational Diffie-Hellman (CDH)

Let *g* be a generator for a cyclic group *G*.



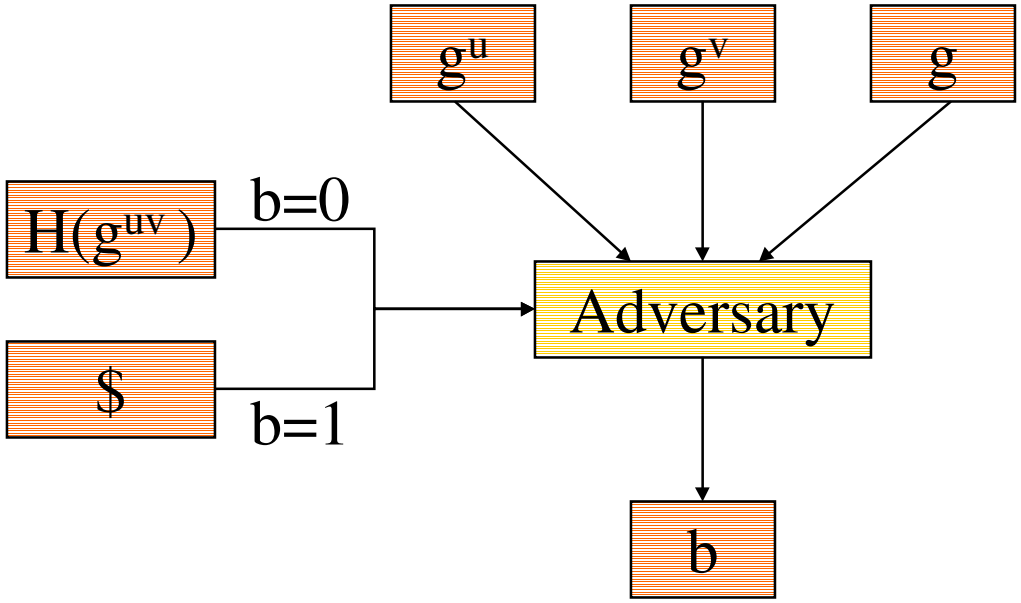
Deciosional Diffie-Hellman (DDH)

Let *g* be a generator for a cyclic group *G* of prime order.



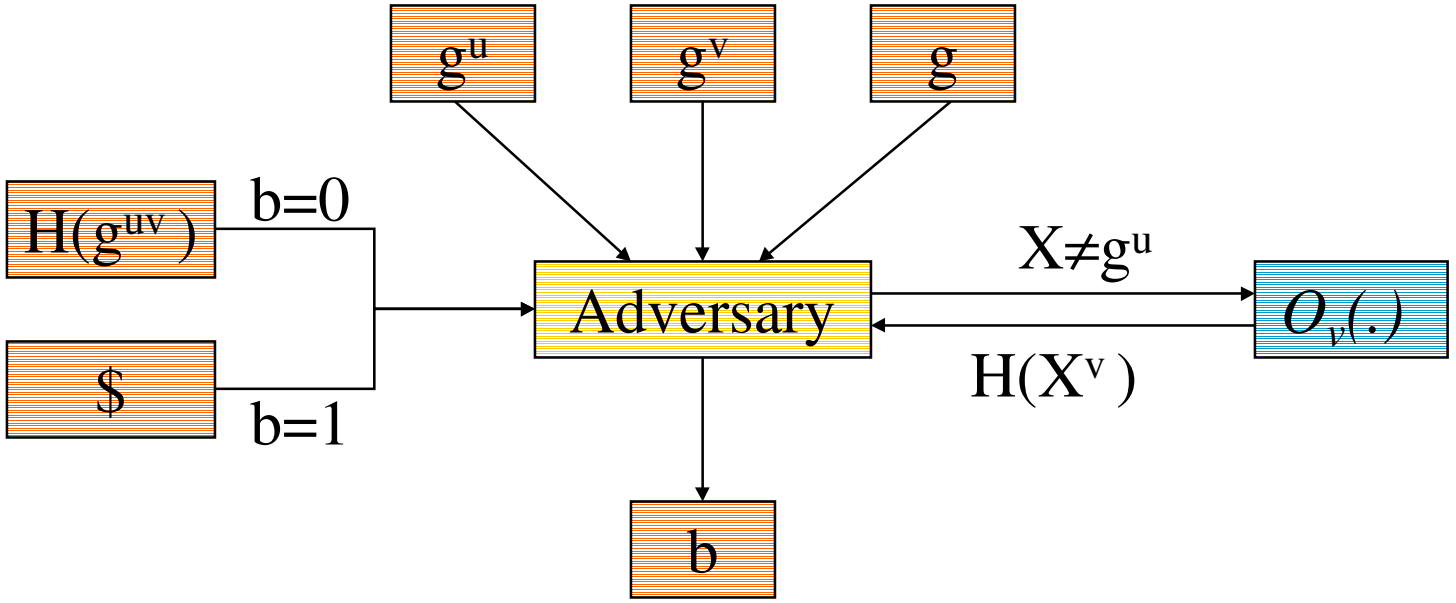
Hash Diffie-Hellman (HDH)

Let *g* be a generator for a cyclic group *G* and *H* a hash function.



Oracle Diffie-Hellman (ODH)

Let O_v be an oracle that, on input X, returns $H(X^v)$.



Rest of the talk

- ✓ Measures of security for public-key encryption schemes
- ✓ Assumptions about hardness of discrete log/Diffie-Hellman related problems
- Security of DHIES

IND-CPA of DHIES

- Theorem: If *HDH* holds in a cyclic group *G* and the underlying *symmetric encryption scheme* is secure, then *DHIES* is IND-CPA secure.

- Quantitative version:

$$\mathrm{Adv}^{\mathrm{ind-cpa}}_{DHIES}(t) \leq$$

$$2 \times \mathrm{Adv}^{\mathrm{hdh}}_{G,H}(t_1) + \mathrm{Adv}^{\mathrm{sym}}_{SYM}(t_2)$$

IND-CCA of DHIES

- Theorem: If *ODH* holds in a cyclic group *G*, the underlying *symmetric encryption scheme* is secure, and the MAC is unforgeable, then *DHIES* is IND-CCA secure.

- Quantitative version:

$$\mathrm{Adv}^{\mathrm{ind\text{-}cca}}_{\mathrm{DHIES}}(t,q) \leq$$

$$2 \cdot \mathrm{Adv}^{\mathrm{odh}}_{G,H}(t_1,q) + 2 \cdot q \cdot \mathrm{Adv}^{\mathrm{mac}}_{MAC}(t_2,q-1) + Adv^{\mathrm{sym}}_{SYM}(t_3)$$

Intuition for IND-CPA of DHIES

Let $C = U \,||EncM\,||Tag$ be the challenge.

- Case 1: Output $H(U^v)$ does not look random.
  Can break the *HDH assumption*.

- Case 2: Output $H(U^v)$ looks random
  Can break the *underlying symmetric encryption*.

Intuition for IND-CCA of DHIES

Let $C = U \,||EncM\,||Tag$ be the challenge.

- Case 1: Output $H(U^v)$ does not look random.

  Can break the *ODH assumption*.

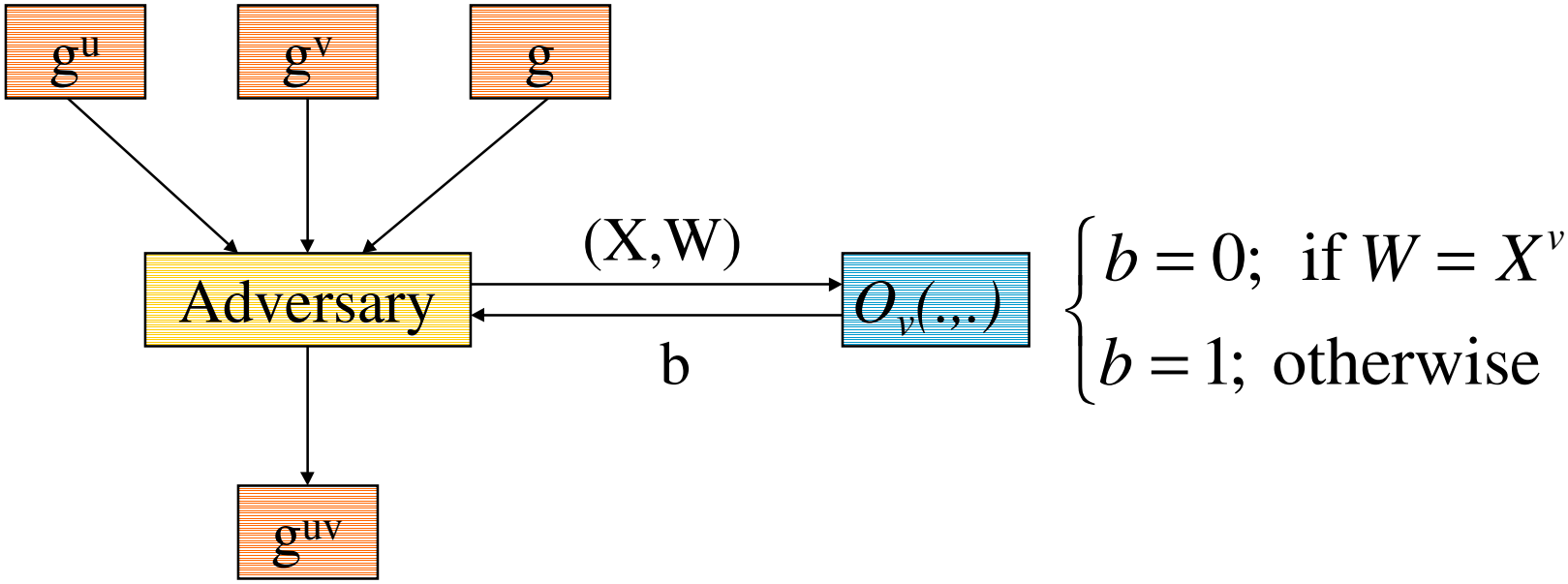
- Case 2: Output $H(U^v)$ looks random
  - 2.1: there is a *valid* query to *decryption oracle*.

    Can break the *underlying MAC*.
  - 2.2: there are no valid queries to *decryption oracle*.

    Can break the *underlying symmetric encryption*.

Proof in Random Oracle Model

- DHIES can alternatively be proven secure in the RO model based on the assumption that following problem (SDH) is hard.



Proof in Generic Model

- In the generic model[Sh97], a lower bound for the hardness of SDH can be proved.
- Theorem: If p is the largest prime factor of the group order, then any generic algorithm solving SDH with non-negligible probability runs in time $O(\sqrt{p})$

Concluding Remarks

- *DHIES* was proven IND-CPA and IND-CCA.
- *DHIES* is as efficient as the basic El Gamal scheme.
- *DHIES* is more efficient than the Cramer-Shoup scheme *[CS98]*, but under stronger assumptions.
- *DHIES* is included in several draft standards: P1363a, ANSI X9.63EC, SEC.