Periodic Binary Sequences with the "Trinomial Property"

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Abstract

Abstract: Periodic binary sequences with the "trinomial property" are considered. Some necessary and sufficient conditions for "trinomial pairs" of a nonlinear sequence of period $2^n - 1$ as well as classifications for trinomial pairs are derived. Complete searches have been done for $3 \le n \le 17$. All trinomial pairs found on this range are listed.

Index Terms: Periodic binary sequence, trinomial property, trinomial pair.

1 Introduction

The m-sequences $A = \{a_i\}$ of degree n and period $p = 2^n - 1$ are characterized by the "cycle-and-add property": for each τ, $0 \le \tau \le p$ there is a corresponding τ', with $\{a_i\} + \{a_{i+\tau}\} = \{a_{i+\tau'}\}$. Thus, the p cyclic shifts of $\{a_i\}$ together with the zero vector of length p form an n-dimensional subspace of F_2^p, the vector space of all p-component vectors over the two-element field F_2.

We say that a binary sequence $A = \{a_i\}$ of period $p = 2^n - 1$ has the *trinomial property* if there is (at least) one pair of positive integers, τ and τ', such that

$$\{a_i\} + \{a_{i+\tau}\} = \{a_{i+\tau'}\} \tag{1}$$

where $0 \le \tau, \tau' \le 2^n - 1$. (Clearly such a sequence has "linear span" $\le \tau'$ if $\tau < \tau'$, and corresponds to the trinomial $x^{\tau'} + x^{\tau} + 1$.)

This paper is organized as follows. In Section 2, we give some necessary and sufficient conditions for finding trinomial pairs for a nonlinear binary sequence of period p. In Sections 3, we discuss classifications for trinomial pairs. In Sections 4 and 5, we list all *transitive* and *intransitive* trinomial pairs, respectively, on the range $3 \le n \le 17$ according to the classifications in Section 3.

2 Some Necessary and Sufficient Conditions

It is known that the sequence $\{a_i\}$ can be written in the following form:

$$a_i = f(\alpha^i),\ i = 0, 1, \cdots, \tag{2}$$

where α is a primitive element of $GF(2^n)$ and $f(x)$ is a function from $GF(2^n)$ to $GF(2)$:

$$f(x) = \sum_{t \in H} Tr_1^{m_t}(\beta_t x^t),\ \beta_t \in GF(2^{m_t})^* \tag{3}$$

where m_t, which is a factor of n, is the degree of the minimal polynomial of α^t, and H is a subset of I, the set of coset leaders of the cyclotomic cosets modulo p. Here we always suppose that $1 \in H$. Let

$$B_{i,t} = Tr_1^{m_t}(\beta_t \alpha^{it}) \text{ and } B_t = \{B_{i,t}\}_{i \geq 0}. \tag{4}$$

Then $A = \sum_{t \in H} B_t$.

Fact 1 *Let $g(x)$ be a function from $GF(2^n)$ to $GF(2)$. Then we have a trace representation for $g(x)$ as follows:*

$$g(x) = \sum_{t \in I} Tr_1^{m_t}(\beta_t x^t),\ \beta \in GF(2^{m_t}) \tag{5}$$

where m_t and I are defined as above. Moreover, $g(x) = 0$ for all $x \in GF(2^n)$ if and only if $\beta_t = 0$ for each $t \in I$.

From this fact, it is clear that

Lemma 1 *A pair (τ, τ') is a trinomial pair of the sequence $\{a_i\}$ if and only if (τ, τ') is a trinomial pair for each component sequence B_t, i.e., $\{a_i\} + \{a_{i+\tau}\} = \{a_{i+\tau'}\}$ if and only if $\{B_{i,t}\} + \{B_{i+\tau,t}\} = \{B_{i+\tau',t}\}$ for each $t \in H$.*

Theorem 1 *A pair (τ, τ') is a trinomial pair of the sequence $\{a_i\}$ if and only if α^τ is a root of*

$$F(x) = (1 + x)^t + 1 + x^t \tag{6}$$

for each $t \in H$ and (τ, τ') satisfies $1 + \alpha^\tau = \alpha^{\tau'}$.

Proof. If (τ, τ') is a trinomial pair of the sequence $\{a_i\}$, we have

$$\begin{aligned} a_i + a_{i+\tau} &= \sum_{t \in H} Tr_1^{m_t}(\alpha^{ti}) + \sum_{t \in H} Tr_1^{m_t}(\alpha^{t(i+\tau)}) \\ &= \sum_{t \in H} Tr_1^{m_t}(\alpha^{ti}(1 + \alpha^{t\tau})). \end{aligned}$$

According to Lemma 1, Condition 1 is equivalent to

$$1 + \alpha^{t\tau} = \alpha^{t\tau'},\ \text{for all } t \in H. \tag{7}$$

Since $1 \in H$, we have

$$1 + \alpha^{\tau} = \alpha^{\tau'}. \tag{8}$$

Consequently

$$(1 + \alpha^{\tau})^t = \alpha^{t\tau'}. \tag{9}$$

From (9), we have

$$F(\alpha) = (1 + \alpha^{\tau})^t + 1 + \alpha^{t\tau} = \alpha^{t\tau'} + 1 + \alpha^{t\tau}.$$

Using (7), it follows that $F(\alpha) = 0$.

□

Corollary 1 *If (τ, τ') is a trinomial pair of the sequence $\{a_i\}$, then $(\tau 2^j, \tau' 2^j)$, $j = 1, \cdots, m_\tau - 1$, are also trinomial pairs of the sequence $\{a_i\}$, where m_τ is the order of τ, which means that m_τ is the smallest positive integer r such that $\tau 2^r \equiv \tau \,(mod\, 2^n - 1)$.*

Definition 1 *Two trinomial pairs (τ, τ') and (τ_1, τ_1') of the sequence $\{a_i\}$ are called* equivalent *if and only if*

$$(\tau_1, \tau_1') \in C_\tau = \{(\tau 2^i, \tau' 2^i) \,|\, i = 0, 1, \cdots, n-1\}.$$

Otherwise, they are called distinct.

From now on, if (τ, τ') is a trinomial pair of the sequence $\{a_i\}$, we will always assume that τ is a coset leader modulo p.

Theorem 2 *If there is one trinomial pair for the sequence $\{a_i\}$, then there are at least two distinct trinomial pairs for $\{a_i\}$.*

To prove Theorem 1, we need the following lemma.

Lemma 2 *Let $\omega \neq 0, 1$ and $\omega \in GF(2^n)$. If ω is a root of $F(x)$, defined by (6), then*

$$\begin{array}{ll} I(\omega) = \omega, & S(\omega) = 1 + \omega, \\ T(\omega) = \omega^{-1}, & V(\omega) = 1 + \omega^{-1} \\ W(\omega) = (1+\omega)^{-1}, & U(\omega) = (1 + \omega^{-1})^{-1} = 1 + W(\omega) \end{array}$$

all are roots of $F(x)$. That is, $F(x)$ is symmetric under the Unitary group of six elements, $G = \{I, S, T, V, W, U\}$ where I is an identity map [2]. In other words, if ω is a root of $F(x)$, then any element in $G\omega = \{g(\omega) | g \in G\}$ is a root of $F(x)$.

Proof. Note that since $F(1 + x) = F(x)$, we only need prove the result for T and W in G. Since $F(\omega) = 0$, we have

$$\begin{aligned} F(T(\omega)) &= F(\omega^{-1}) \\ &= (1 + \omega^{-1})^t + 1 + \omega^{-t} \\ &= \omega^{-t}(1 + \omega)^t + 1 + \omega^{-t} \end{aligned}$$

$$\begin{aligned} &= \omega^{-t}(1+\omega)^t + 1 + \omega^t) \\ &= \omega^{-t}F(\omega) = 0; \\ F(W(\omega)) &= F((1+\omega)^{-1}) \\ &= (1+(1+\omega)^{-1})^t + 1 + (1+\omega)^{-1} \\ &= (1+\omega)^{-1}F(\omega) = 0. \end{aligned}$$

□

Proof of Theorem 2. Let (τ, τ') be a trinomial pair of the sequence $\{a_i\}$ and let $\omega = \alpha^\tau$. Then $\omega \neq 0$ and 1. According to Lemma 2, the other five elements generated by the unitary group G are all roots of $F(x)$. Let

$$K(\omega) = \alpha^{\tau_K}, K \in G.$$

Since α is a primitive element of $GF(2^n)$, there exist positive integers τ'_K such that $1 + \alpha^{\tau_K} = \alpha^{\tau'_K}$ for $K \in G$. From Theorem 1, the (τ_K, τ'_K)'s are five other trinomial pairs of $\{a_i\}$.

Case 1. n is an odd number.

In this case, ω and $T(\omega)$ have different minimal polynomials [2], so $(\tau_T, \tau'_T) \notin C_\tau$. Therefore (τ, τ') and (τ_T, τ'_T) are distinct trinomial pairs. So, if $\{a_i\}$ has one trinomial pair, then it has at least two distinct trinomial pairs.

Case 2. n is an even number.

If all six elements have the same minimal polynomial, according to [2] the order of $T(\omega)$ is 2^k+1 where $n = 2k$. On the other hand, from

$$S(\omega) = 1 + \omega = \omega^{2^i} \text{ and } V(\omega) = 1 + \omega^{-1} = \omega^{2^j},$$

we have $\omega^{2^j+1} = \omega^{2^i}$. This implies $2^j + 1 \equiv 2^i \pmod{2^k+1}$, which is impossible. So, there are at least two of these elements having different minimal polynomials. Thus $\{a_i\}$ has at least two distinct trinomial pairs.

□

Remark 1 *From the proofs of Lemma 2 and Theorem 2, we know that if (τ, τ') is a trinomial pair of $\{a_i\}$, so is (τ', τ).*

From [2] and the argument of the proof of Theorem 2, we know that for the elements in $G\omega$, either each one of them or each pair of them, or each trio of them have different minimal polynomials. So, Theorem 2 can be stated more precisely as

Corollary 2 *If there is one trinomial pair for the sequence $\{a_i\}$, then there are at least two trinomial pairs, and possiblely three pairs, or six pairs, for $\{a_i\}$.*

Theorem 3 *Let m be a divisor of n, and let $d = (2^n-1)/(2^m-1)$. If $t \equiv 2^i \pmod{2^m-1}$ for each $t \in H$, then $\{a_i\}$ has all the following pairs as its trinomial pairs:*

$$(\tau d, \tau' d) : 1 \le \tau \le 2^m - 1$$

where $1 + \alpha^{\tau d} = \alpha^{\tau' d}$.

Proof. According to Theorem 1, we only need prove that $\alpha^{\tau d}, 1 \leq \tau \leq 2^m - 1$, are roots of $F(x)$. Notice that α^d is a primitive element of $GF(2^m)$, so $\alpha^{\tau d}$ and $1 + \alpha^{\tau d} \in GF(2^m)$. Therefore the exponents of $\alpha^{\tau d}$ and $1 + \alpha^{\tau d}$ can be reduced modulo $2^m - 1$. Since $t \equiv 2^i \pmod{2^m - 1}$, it follows that

$$\begin{aligned} F(\alpha^{\tau d}) &= (1+\alpha^{\tau d})^t + 1 + (\alpha^{\tau d})^t \\ &= (1+\alpha^{\tau d})^{2^i} + 1 + (\alpha^{\tau d})^{2^i} = 0. \end{aligned}$$

The result follows immediately.

□

Definition 2 *The trinomial pairs given by Theorem 3 are called the* regular trinomial pairs *of* $\{a_i\}$*; other trinomial pairs are called* non-regular trinomial pairs *of* $\{a_i\}$*, or simply,* trinomial pairs *of* $\{a_i\}$*.*

From the discussion of linear spans of GMW sequences and cascaded GMW sequences in references [5] and [4], we have the following corollary.

Corollary 3 *Any GMW sequence or cascaded GMW sequence has regular trinomial pairs.*

3 Classifications and Experimental Results

For all $n > 3$, there exist non-regular trinomial pairs. We have done complete searches for $3 \leq n \leq 17$. For greater convenience in presenting our experimental results, we introduce some new terms.

Definition 3 *If* $t \in I$*, where* I *is the set of coset leaders modulo* $2^n - 1$*, and* t *is not congrent to* 2^i *modulo* $2^m - 1$ *for any proper factor* m *of* n*, then a function* $Tr_1^{m_t}(x^t)$ *from* $GF(2^n)$ *to* $GF(2)$ *is called a* complete non-linear (CNL) function *and* t *is called a* complete non-linear (CNL) exponent*. Otherwise it is called an* incomplete non-linear (ICNL) function *and* t *is called an* incomplete non-linear (ICNL) exponent*.*

From Theorem 3, we know that a sequence generated by a sum of ICNL functions has regular trinomial pairs.

For t and $\tau \in I$, we define two maps on $GF(2^n)$ as follows,

$$\begin{aligned} \text{“Add 1 map”} &: \quad A(\alpha) = 1 + \alpha; \\ \text{“ Power map”} &: \quad P_t(\alpha) = \alpha^t. \end{aligned}$$

Notice that

$$P_t A P_\tau(\alpha) = (1+\alpha^\tau)^t; \tag{10}$$

$$A P_t P_\tau(\alpha) = 1 + \alpha^{t\tau}, \tag{11}$$

and

$$P_t P_\tau = P_{t\tau}.$$

From Theorem 1, (τ, τ') is a trinomial pair of the component sequence B_t, defined by (4), if and only if

$$P_t A P_\tau(\alpha) = A P_{t\tau}(\alpha). \tag{12}$$

Let $I_{CNL} = \{t \in I | \, t \text{ is CNL}\}$ and $I_{ICNL} = \{t \in I | \, t \text{ is ICNL}\}$. Then $I = I_{CNL} \cup I_{ICNL}$ and $I_{CNL} \cap I_{ICNL} = \emptyset$. For a fixed $t \in I$, define

$$\Lambda_t = \{(\tau, \tau') | P_t A P_\tau(\alpha) = A P_{t\tau}(\alpha)\}. \tag{13}$$

For a fixed pair (τ, τ'), define

$$\Phi_\tau = \{t \,|\, P_t A P_\tau(\alpha) = A P_{t\tau}(\alpha)\}. \tag{14}$$

Let

$$\Phi_{\tau,CNL} = \{t \in \Phi_\tau | \, t \in I_{CNL}\}, \tag{15}$$

and

$$\Phi_{\tau,ICNL} = \{t \in \Phi_\tau | \, t \in I_{ICNL}\}; \tag{16}$$

then

$$\Phi_\tau = \Phi_{\tau,CNL} \cup \Phi_{\tau,ICNL}.$$

Definition 4 *For a fixed pair (τ, τ'), let $\Phi_\tau \neq \emptyset$, and let $\{a_i\}$ satisfy $H \subseteq \{1\} \cup \Phi_\tau$. If both $\Phi_{\tau,CNL}$ and $\Phi_{\tau,ICNL}$ are nonempty, then (τ, τ') is called a* transitive (non-regular) trinomial pair (TTP) *of $\{a_i\}$. If one of $\Phi_{\tau,CNL}$ and $\Phi_{\tau,ICNL}$ is empty, then (τ, τ') is called an* intransitive (non-regular) trinomial pair (ITTP) *of $\{a_i\}$.*

The experimental results suggested the following conjectures.

Conjecture 1 *For $t \in I$ and $t \neq 1$, if n is a prime, then $\Lambda_t = \emptyset$. In other words, if n is a prime, then any non-linear binary sequence of period $2^n - 1$ has no trinomial pairs.*

The validity of this conjecture was verified for $3 \leq n \leq 17$. I.e., for $n = 3, 5, 7, 11, 13$, and 17, any nonlinear binary sequence of period $p = 2^n - 1$ has no trinomial pairs.

Conjecture 2 *(Duality) Let $t \neq r$, where t and r are both either in I_{CNL} or in I_{ICNL}. Let (τ, τ') and (ρ, ρ') be two pairs. Then*

$$\begin{aligned} (\tau, \tau') \in \Lambda_t \cap \Lambda_r \quad &\Longrightarrow \quad \Lambda_t = \Lambda_r; \\ t \in \Phi_\tau \cap \Phi\rho \quad &\Longrightarrow \quad \Phi_\tau = \Phi\rho. \end{aligned}$$

The validity of Conjecture 2 was verified for $3 \leq n \leq 17$.

4 Transitive Trinomial Pairs for $3 \leq n \leq 17$

In this section, we list all transitive trinomial pairs for $3 \leq n \leq 17$. For two trinomial pairs (τ, τ') and (τ', τ), we only list one of them and the first component is a coset leader modulo p for each pair. This rule also applies to the next section. Note that in Conjecture 2, if t and r are not both CNL or ICNL, Conjecture 2 is still true for $n = 8, 9, 12, 15$. For $n = 16$, where the first counter-examples occurs, we list them separately in Table 9. For example, from Table 9, we have $\Lambda_{1329} = \{(3825, 35965), (7967, 11565)\}$ where 1329 is a CNL exponent and $\Lambda_{1295} = \{(3825, 35965)\}$ where 1295 is an ICNL exponent, even though $(3825, 35965) \in \Lambda_{1329} \cap \Lambda_{1295}$, $\Lambda_{1329} \neq \Lambda_{1295}$.

Table 1 $n = 8$

$n = 8$ and $\alpha^8 + \alpha^4 + \alpha^3 + \alpha^2 + 1 = 0$		
t, CNL	t, ICNL	(τ, τ'), TTP
87	43	(9, 120) (111, 246)

Table 2 $n = 9$

$n = 9$ and $\alpha^9 + \alpha^4 + 1 = 0$		
t, CNL	t, ICNL	(τ, τ'), TTP
55, 75, 77, 223	37, 183	(49, 119)

Table 3 $n = 10$

$n = 10$ and $\alpha^{10} + \alpha^3 + 1 = 0$		
t, CNL	t, ICNL	(τ, τ'), TTP
123	53	(31, 758) (37, 161)
183	251	(85, 434) (343, 746)

Table 4 $n = 12$

$n = 12$ and $\alpha^{12} + \alpha^6 + \alpha^4 + \alpha + 1 = 0$		
t, CNL	t, ICNL	(τ, τ'), TTP
117 , 507, 663	137, 239, 275, 277 281, 637, 751, 1775 1919	(885, 2760)
411	41, 235, 733	(197, 323) (943, 2016)

Table 5 $n = 14$

$n = 14$ and $\alpha^{14} + \alpha^5 + \alpha^3 + \alpha + 1 = 0$		
t, CNL	t, ICNL	(τ, τ'), TTP
51, 3435, 392	745, 959, 1783 2357, 2431	(889, 7991)
219, 717, 2643	619, 751, 1171 1351, 1507	(979, 6731)
45 , 573, 1467	785, 1973, 2555 2765, 3047	(1397, 13663)
1209, 2781	101, 461, 1003 1915	(381, 1803) (1959, 5007)
1179, 1353	415, 1469, 3407 3839	(945, 11049) (1263, 14217)
579, 1197	77, 755, 3803 5615	(1143, 8247) (1017, 15495)
147, 741, 1977	227, 491, 859 2653, 3059	(883, 2540) (1657, 15500)
201, 1959, 4053	361, 1757, 3035 3319, 5623	(1003, 13703) (1651, 12371)
1641, 2919, 4077	479, 1001, 1607 1781, 3023	(1193, 5257) (5563, 14351)

Table 6 $n = 15$

$n = 15$ and $\alpha^{15} + \alpha + 1 = 0$								
t, CNL				t, ICNL				(τ, τ'), TTP
173,	251,	327,	633	151,	529,	793,	845	(5487, 21018)
951,	1475,	1711,	1749	925,	991,	1059,	1061	
2287,	2323,	2959,	3517	1065,	1073,	1359,	1403	
3673,	4843,	4861,	5631	1661,	2265,	2643,	2729	
5787,	5939,	6975,	7901	2907,	3039,	3173,	3175	
8053				3179,	3187,	3775,	4007	
				4757,	5021,	5287,	5293	
				5301,	5587,	6871,	7135	
				7403,	7407,	7415,	11099	
				11631,	11643,	15327,	15871	

Table 7 $n = 15$

$n = 15$ and $\alpha^{15} + \alpha + 1 = 0$				
t, CNL	t, ICNL	(τ, τ'), TTP		
1207, 15855	2791, 10999	(199, 16323)	(4031, 8142)	
2939	411, 1143	(815, 22880)	(5351, 21327)	(3179, 13374)
1013, 5031	855	(1571, 31120)	(1609, 17207)	(7023, 20199)
1587, 4045	2727	(885, 5150)	(11119, 28907)	(2809, 10529)

Table 8 $n = 16$ **and** $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$

t, CNL	t, ICNL	(τ, τ'), TTP
147, 5709, 6351 11487, 13695	247, 2473, 2701, 3055 3895, 4427, 4601, 6011 9559, 15083	(765, 16445)
5331, 7791, 7929 10167, 11895	1945, 3197, 3397, 3731 3785, 4559, 5021, 6601 7757, 28639	(255, 12849) (14139, 15159)
1569, 1737, 2877 4443 , 5715	367, 1015, 1171, 1643 1943, 3259, 3445, 5947 11707, 15791	(381, 32001) (1533, 2295)
1323, 2163, 5241 5517 , 6105	181, 223, 1925, 3001 4951, 7603, 7673, 11885 13543, 22207	(1743, 14280) (3987, 37647)
1497, 1929, 4461 4767, 11445	1045, 1519, 1889, 2459 6617, 6653, 9973, 11671 11887, 23935	(2805, 29544) (6447, 12057)
219, 909, 1437 9885	2419, 2863, 3235, 4595 5273, 6487, 7763, 8171 11771, 24559	(3685, 25870) (10965, 24115)
2247, 7413, 7785 9837, 12147	2449, 2543, 3631, 3821 4477, 5065, 6967, 7807 9383, 14815	(4521, 43281) (4845, 56778)
327, 3693, 9429 9915, 13287	535, 649, 1249, 1525 6125, 7529, 12127, 15031 23287	(5087, 57107) (11475, 45187)
1053, 1389, 2547 7335, 8181	947, 1001, 1271, 2375 4805, 4907, 9629, 11255 14173	(5355, 9488) (24059, 64349)
2223, 3369, 5007 10215, 11061	2029, 2675, 4589, 5995 6139, 7615, 8087, 9883 10751	(9435, 10243) (13823, 65333)
207, 981, 4425 5229, 7119, 9399	601, 1303, 2705, 3773 4895, 6365, 14261, 15071 16213	(1275, 15158) (4903, 10003)
1641, 2001, 2853 3477, 5583, 6975	785, 1325, 3517, 5291 6431, 7591, 10069, 12251 14335	(3315, 54728) (7061, 38171)
699, 813, 4503 5751, 7113, 24315	109, 1559, 2603, 3671 4775, 5071, 7093, 7775 12275	(6375, 8456) (31711, 57211)

Table 9 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$	
t, CNL	(τ, τ'), TTP
1329, 3207, 8157, 10875 11991	(3825, 35965) (11565, 7967)
3237, 3675, 4407, 10071 28383	(6885, 61139) (11565, 7967)
t, ICNL	(τ, τ'), TTP
1295, 2119, 5077, 5731 6527, 7613, 9419, 9715 11263, 22199	(3825, 35965)
725, 1013, 1487, 2855 3755, 4031, 4843, 6127 9391	(6885, 61139)

5 Intransitive Trinomial Pairs for $3 \leq n \leq 17$

For each n, we list intransitive trinomial pairs for CNL exponents following by ICNL exponents. If exponents whose orders are less n, we will indicate them. For all others, they have order n.

Table 10 $n = 6$

$n = 6$ and $\alpha^6 + \alpha + 1 = 0$	
t, CNL	(τ, τ')
21, order 2	(11, 25) (13, 35)

Table 11 $n = 8$

$n = 8$ and $\alpha^8 + \alpha^4 + \alpha^3 + \alpha^2 + 1 = 0$	
t, CNL	(τ, τ')
27	(7, 112) (31, 45)
t, ICNL	(τ, τ')
85, order 2	(19, 92) (29, 181) (37, 179)

Table 12 $n = 9$

$n = 9$ and $\alpha^9 + \alpha^4 + 1 = 0$			
t, CNL	(τ, τ')		
73, order 3	(27, 142)	(51, 454)	(79, 307)
t, ICNL	(τ, τ')		
219, order 3	(19, 421)	(45, 310)	(75, 435)

Table 13 $n = 10$

$n = 10$ and $\alpha^{10} + \alpha^3 + 1 = 0$				
t, CNL	(τ, τ')			
99, order 5	(47, 397)	(53, 595)	(61, 917)	
165, order 5	(35, 149)	(57, 494)	(111, 854)	
t, ICNL	(τ, τ')			
171, 343	(375, 393)	(447, 723)	(69, 144)	(93, 657)
	(183, 741)			
341, order 2	(13, 314)	(19, 767)	(31, 758)	(37, 161)
	(47, 397)	(53, 595)	(61, 917)	(1, 77)
	(85, 434)	(91, 299)	(181, 971)	(187, 251)
	(343, 746)			

Table 14 $n = 12$

$n = 12$ and $\alpha^{12} + \alpha^6 + \alpha^4 + \alpha + 1 = 0$				
t, CNL	(τ, τ')			
21, 105,147, 231 357, 399, 735, 861 987, 1407	(455, 2860)	(845, 2665)		
195	(615, 2748)	(885, 2760)	(171, 2865)	(213, 2718)
273, order 4	(411, 3343)	(621, 3913)	(91, 2449)	(153, 2009)
	(197, 323)	(361, 3589)	(77, 383)	(455, 2860)
	(603, 2969)	(627, 2651)	(845, 2665)	(29, 2493)
	(943, 2016)			
1911, order 4	(339, 3962)	(441, 1943)	(1371, 2813)	(1503, 2134)
	(113, 1071)	(133, 472)	(209, 1091)	(283, 1327)
	(301, 597)	(319, 1383)	(455, 2860)	(479, 1991)
	(631, 2592)	(749, 2066)	(845, 2665)	
1755, order 3	(495, 1786)	(507, 373)	(615, 2748)	(885, 2760)
	(89, 3914)	(135, 2138)	(145, 4082)	(157, 2221)
	(167, 4071)	(169, 180)	(171, 2865)	(191, 24)
	(213, 2718)	(237, 1439)	(251, 1915)	(303, 2852)
	(349, 193)	(379, 2112)	(491, 488)	(629, 3966)
	(765, 3391)	(877, 2089)	(1007, 2496)	(1391, 176)
1365, order 2	(97, 107)	(101, 2170)	(109, 3587)	(121, 377)
	(127, 1178)	(139, 1451)	(145, 4082)	(149, 2851)
	(151, 716)	(175, 3407)	(221, 4093)	(1, 2159)
	(229, 3359)	(233, 3490)	(235, 3179)	(251, 1915)
	(277, 1226)	(299, 3286)	(311, 1372)	(313, 1139)
	(335, 2059)	(367, 2972)	(371, 814)	(397, 1865)
	(413, 1891)	(425, 2887)	(455, 2860)	(469, 3422)
	(607, 731)	(661, 3767)	(671, 3775)	(695, 3316)
	(845, 2665)	(43, 3893)	(949, 3818)	(41, 2683)
	(5, 1999)			

Table 15 $n = 12$

$n = 12$ and $\alpha^{12} + \alpha^6 + \alpha^4 + \alpha + 1 = 0$				
t, ICNL	(τ, τ')			
103, 605, 939, 1471	(113, 1071)	(479, 1991)		
185, 219, 397, 499	(441, 1943)	(209, 1091)		
91	(363, 2757)	(693, 3369)	(885, 2760)	
715	(951, 2105)	(291, 521)	(415, 3949)	
1495	(957, 535)	(177, 1657)	(211, 1780)	
293, 439, 587, 589 1463, 1759	(987, 119)	(147, 623)	(217, 994)	
455	(381, 636)	(885, 2760)	(93, 4065)	(15, 2109)
205, 821, 823, 827	(315, 3505) (275, 3710)	(885, 2760) (295, 2500)	(1455, 2995)	(165, 1120)
683, 1367	(297, 1158) (423, 1266) (735, 1566) (93, 4065) (213, 2718)	(357, 3291) (615, 2748) (831, 3360) (123, 30) (219, 2892)	(363, 2757) (693, 3369) (861, 3798) (171, 2865)	(381, 636) (717, 1593) (885, 2760) (207, 102)
585, order 3	(327, 403) (121, 377) (293, 3089) (627, 2651) (989, 328)	(411, 3343) (139, 1451) (309, 3811) (661, 3767) (991, 320)	(699, 2998) (221, 4093) (361, 3589) (671, 3775)	(97, 107) (223, 2) (471, 2057) (871, 3487)

Table 16 $n = 14$

$n = 14$ and $\alpha^{14} + \alpha^5 + \alpha^3 + \alpha + 1 = 0$				
t, CNL	(τ, τ')			
2451	(767, 8671)			
1701	(219, 7850)	(2367, 13277)		
207	(49, 6252)	(2511, 8783)	(3839, 15200)	
387	(435, 10220)	(2463, 1843)	(1639, 4960)	
645	(615, 2456)	(1971, 2278)	(1655, 13118)	
1161	(2379, 2699)	(5, 13530)	(3421, 16303)	
1419	(1095, 15509)	(1911, 14089)	(1147, 10294)	
1947	(1383, 10736)	(1875, 3217)	(3515, 11015)	
3741	(803, 1717)	(1759, 5294)	(3535, 14624)	
3933	(603, 3152)	(2541, 10831)	(3775, 6187)	
7095	(459, 6343)	(1695, 8075)	(1255, 7456)	
903, order 7	(531, 12239)	(1197, 3674)	(2997, 13466)	(2927, 3963)
	(841, 12352)	(1687, 6475)		
1677, order 7	(351, 12273)	(501, 9588)	(793, 1610)	(817, 15590)
	(1461, 57)	(3311, 7006)		
2709, order 7	(97, 9057)	(2535, 4087)	(1175, 9325)	(1901, 13306)
	(2359, 5312)	(35, 10175)		
2967, order 7	(273, 8997)	(477, 2454)	(567, 13134)	(657, 7647)
	(1119, 14115)	(3495, 7863)	(1097, 14716)	(3317, 11707)
6063, order 7	(1251, 5201)	(1851, 7055)	(853, 12947)	(859, 5168)
	(2735, 13291)	(5591, 14408)		

Table 17 $n = 14$

$n = 14$ and $\alpha^{14} + \alpha^5 + \alpha^3 + \alpha + 1 = 0$				
t, ICNL	(τ, τ')			
2771	(871, 16193)	(1939, 6011)		
151	(1125, 9242)	(3447, 6847)	(2783, 5378)	
217	(789, 8302)	(3759, 5527)	(1357, 16162)	
761	(1317, 14237)	(5847, 5062)	(785, 11321)	
1141	(1437, 2597)	(1491, 4132)	(5819, 16238)	
1247	(351, 12273)	(501, 9588)	(57, 1461)	
5599	(983, 9925)	(1117, 3737)	(1501, 14417)	
473	(273, 8997)	(477, 2454)	(567, 13134)	(657, 7647)
	(1119, 14115)	(3495, 7863)		
2731	(177, 13659)	(195, 1728)	(243, 13719)	(267, 15804)
5463	(273, 8997)	(333, 6507)	(339, 4905)	(351, 12273)
	(363, 13002)	(381, 1803)	(417, 7245)	(423, 8481)
	(429, 5997)	(471, 12585)	(477, 2454)	(489, 6516)
	(501, 9588)	(567, 13134)	(591, 13503)	(657, 7647)
	(681, 4821)	(687, 16245)	(693, 15192)	(723, 11910)
	(735, 12651)	(807, 987)	(69, 8604)	(831, 10548)
	(843, 7470)	(915, 6705)	(933, 13404)	(939, 13992)
	(945, 11049)	(981, 7116)	(1119, 14115)	(1143, 8247)
	(1173, 4605)	(1179, 5115)	(1221, 6036)	(1263, 14217)
	(1311, 10839)	(1335, 6870)	(1359, 12252)	(1407, 14931)
	(1533, 16188)	(1707, 12183)	(1743, 10725)	(1881, 15027)
	(1893, 2727)	(1947, 8214)	(1959, 5007)	(1995, 5103)
	(2391, 3330)	(123, 2523)	(2559, 4119)	(2655, 14346)
	(2733, 13551)	(2877, 11499)	(3381, 3744)	(3495, 7863)
	(1017, 15495)	(57, 1461)		

Table 18 $n = 14$

$n = 14$ and $\alpha^{14} + \alpha^5 + \alpha^3 + \alpha + 1 = 0$				
t, ICNL	(τ, τ')			
5461	(751, 4964)	(755, 9442)	(763, 13748)	(767, 8671)
order 2	(89, 787)	(793, 1610)	(803, 1717)	(817, 15590)
	(827, 11350)	(829, 15329)	(839, 16252)	(851, 9625)
	(853, 12947)	(859, 5168)	(863, 2959)	(871, 16193)
	(883, 2540)	(889, 7991)	(895, 5735)	(905, 13255)
	(931, 13385)	(943, 11729)	(953, 2389)	(955, 12212)
	(965, 2722)	(977, 5383)	(979, 6731)	(983, 9925)
	(1003, 13703)	(1015, 4076)	(1097, 14716)	(1103, 16282)
	(1117, 3737)	(1135, 6593)	(1141, 7508)	(1165, 14681)
	(1175, 9325)	(1193, 5257)	(1195, 12353)	(625, 1223)
	(1267, 7076)	(1307, 2455)	(1331, 15778)	(1337, 10174)
	(1367, 3649)	(1375, 3545)	(1379, 8182)	(1397, 13663)
	(1399, 12209)	(697, 1433)	(1453, 3713)	(1465, 5354)
	(1481, 15274)	(1487, 2668)	(1499, 10156)	(1501, 14417)
	(1519, 11396)	(1525, 11507)	(1531, 7343)	(191, 8959)
	(1589, 2857)	(311, 4498)	(1651, 12371)	(1657, 15500)
	(1685, 15331)	(1741, 8048)	(1759, 5294)	(709, 9077)
	(1781, 8203)	(1853, 12127)	(481, 13745)	(1895, 4021)
	(1901, 13306)	(251, 12763)	(1915, 11012)	(1939, 6011)
	(1963, 14411)	(2003, 5557)	(2009, 6667)	(2027, 10537)
	(2039, 6721)	(2045, 6586)	(2341, 7172)	(2359, 5312)
	(2365, 7658)	(2395, 6146)	(2429, 7027)	(2485, 3701)
	(2519, 14371)	(2653, 6788)	(2735, 13291)	(2741, 5665)
	(541, 4787)	(2773, 11309)	(2791, 13331)	(2815, 8087)
	(2899, 7130)	(2903, 10870)	(2923, 12128)	(2941, 7019)
	(2999, 13132)	(3055, 3947)	(3061, 15368)	(3311, 7006)
	(3317, 11707)	(3391, 8543)	(3535, 14624)	(3581, 13333)
	(3773, 11296)	(3931, 4355)	(493, 13271)	(5563, 14351)
	(133, 15038)	(6079, 15680)	(29, 12253)	

Table 19 $n = 15$

$n = 15$ and $\alpha^{15} + \alpha + 1 = 0$				
t, CNL	(τ, τ')			
623	(7671, 16397)	(5119, 19934)		
139	(465, 1009)	(479, 15359)	(3007, 2049)	
209	(651, 28857)	(2999, 23870)	(5487, 21018)	
647	(2197, 2561)	(5117, 32491)	(11195, 12299)	
3191	(829, 30303)	(3293, 2464)	(2969, 3046)	
3727	(1085, 13738)	(1519, 23237)	(4765, 21908)	
7839	(217, 16872)	(1007, 15329)	(1985, 4991)	
2365	(3255, 4472)	(3689, 4248)	(5487, 21018)	(8059, 17390)
3065	(775, 25166)	(2523, 16907)	(3965, 29171)	(5487, 21018)
1497	(1147, 23249)	(3253, 1131)	(5547, 24893)	(12155, 22324)
	(4759, 21716)	(5367, 18960)		
1655	(2015, 21107)	(3069, 21169)	(3433, 26629)	(4773, 7688)
	(2915, 27994)	(5303, 21733)		
615	(923, 6038)	(1693, 16461)	(2463, 25358)	(617, 2511)
	(5115, 31844)	(5487, 21018)	(5883, 26126)	
2747	(997, 3446)	(2035, 14120)	(2201, 22465)	(2533, 28396)
	(5487, 21018)	(6831, 1656)	(3449, 3242)	

Table 20 $n = 15$

$n = 15$ and $\alpha^{15} + \alpha + 1 = 0$				
t, CNL	(τ, τ')			
1057	(173, 28255)	(483, 5067)	(573, 20419)	(885, 5150)
order 5	(955, 18765)	(1615, 10982)	(1777, 4954)	(1949, 15699)
	(2033, 28658)	(2285, 16180)	(2359, 14361)	(2709, 29203)
	(2925, 6819)	(3177, 30990)	(3241, 21270)	(3801, 9692)
	(4709, 11041)	(4723, 26969)	(5851, 6133)	(6839, 13164)
	(7005, 11419)	(7675, 21931)	(11119, 28907)	(2809, 10529)
	(2853, 30857)	(3071, 18438)	(141, 13458)	(4701, 26307)
	(5307, 29404)	(6875, 15409)		
3171	(147, 10016)	(299, 11064)	(367, 29576)	(633, 10479)
order 5	(689, 31129)	(819, 17547)	(1109, 16738)	(1119, 8486)
	(1393, 19864)	(1475, 21192)	(1531, 7346)	(1655, 22505)
	(1657, 9279)	(1725, 21883)	(1739, 2627)	(1977, 14723)
	(3191, 3558)	(3405, 27902)	(3421, 5712)	(3667, 16951)
	(3699, 16878)	(3923, 25943)	(5815, 31236)	(2965, 19527)
	(3175, 7131)	(3321, 7275)	(3369, 23657)	(3579, 5931)
	(3805, 28585)	(3901, 16742)	(4923, 16067)	(4943, 14208)
	(6099, 27066)	(7535, 32545)	(7549, 29317)	(8117, 10883)
5285	(153, 16061)	(271, 20038)	(579, 2474)	(787, 15926)
order 5	(945, 8177)	(645, 951)	(1181, 4927)	(1743, 21598)
	(1827, 27221)	(1939, 31960)	(2799, 26118)	(97, 10426)
	(3275, 25459)	(3531, 5699)	(3545, 13287)	(59, 3839)
	(5997, 26332)	(6767, 32225)	(2909, 19806)	(7903, 20773)
	(2447, 25950)	(915, 2489)	(113, 7665)	(2773, 20070)
	(1873, 15793)	(3273, 16772)	(2335, 22975)	(7935, 22864)
	(8047, 25049)			
7399	(383, 31034)	(1133, 12359)	(1347, 26075)	(1501, 29586)
order 5	(1673, 26585)	(2467, 9671)	(2551, 23172)	(1675, 2747)
	(3181, 4682)	(3727, 10015)	(3951, 26850)	(4021, 21861)
	(4073, 21589)	(4731, 31218)	(4983, 10839)	(5453, 23847)
	(5967, 17152)	(7615, 25474)	(7647, 13843)	(2517, 8175)
	(253, 15343)	(4855, 23127)	(1159, 19357)	(611, 27379)
	(2237, 28022)			

Table 21 $n = 15$

$n = 15$ and $\alpha^{15} + \alpha + 1 = 0$				
t, CNL	(τ, τ')			
4681	(129, 5097)	(131, 12317)	(201, 17560)	(229, 9531)
order 3	(271, 20038)	(299, 11064)	(321, 32174)	(325, 26517)
	(339, 29261)	(367, 29576)	(377, 23921)	(465, 1009)
	(479, 15359)	(521, 22191)	(661, 19125)	(699, 24005)
	(731, 4859)	(747, 17602)	(787, 15926)	(943, 31273)
	(1081, 26384)	(1109, 16738)	(1119, 8486)	(1147, 23249)
	(1161, 14513)	(1195, 24763)	(1223, 5653)	(1235, 16472)
	(1245, 9102)	(1259, 7786)	(1277, 30689)	(1293, 28382)
	(1363, 3014)	(1371, 8829)	(1389, 6973)	(1567, 10600)
	(1623, 6659)	(1637, 22507)	(1657, 9279)	(1679, 30340)
	(1699, 6899)	(1725, 21883)	(1741, 24966)	(1753, 18306)
	(781, 15118)	(1853, 12702)	(1909, 20500)	(1931, 28856)
	(1951, 32365)	(1963, 30276)	(2001, 26875)	(2007, 30055)
	(2015, 21107)	(2215, 15772)	(2231, 2314)	(2281, 5077)
	(2411, 21358)	(2427, 4106)	(2491, 4454)	(2539, 6066)
	(2223, 2553)	(2603, 20470)	(2617, 4125)	(2687, 5763)
	(2805, 19240)	(2911, 28443)	(2943, 28624)	(2967, 27561)
	(3069, 21169)	(3191, 3558)	(1131, 3253)	(3365, 26499)
	(3433, 26629)	(3435, 19562)	(3527, 30228)	(3531, 5699)
	(3545, 13287)	(3645, 9307)	(3699, 16878)	(3741, 6063)
	(3797, 10410)	(3827, 21123)	(3923, 25943)	(4773, 7688)
	(1871, 4829)	(379, 29298)	(5071, 25768)	(65, 19123)
	(5547, 24893)	(6093, 16318)	(23, 20271)	(6131, 12623)
	(6613, 28970)	(6621, 21744)	(6767, 32225)	(6893, 31119)
	(6999, 12070)	(7643, 16865)	(7677, 27171)	(7853, 27421)
	(1989, 22375)	(2739, 21867)	(443, 11647)	(1943, 12031)
	(12143, 26618)	(12155, 22324)	(16127, 29332)	(915, 2489)
	(2531, 4093)	(2761, 25439)	(2915, 27994)	(17, 15345)
	(3061, 12466)	(3175, 7131)	(481, 20913)	(75, 20899)
	(3519, 22495)	(3817, 12298)	(4759, 21716)	(4853, 20523)
	(5303, 21733)	(5333, 16095)	(5367, 18960)	(5755, 17083)
	(5847, 21237)	(7549, 29317)	(7851, 24988)	(535, 27294)
	(8117, 10883)	(621, 28655)		

Table 22 $n = 15$

$n = 15$ and $\alpha^{15} + \alpha + 1 = 0$				
t, ICNL	(τ, τ')			
1335	(3727, 10015)	(7615, 25474)		
3403	(2525, 20285)	(8053, 16993)		
1893	(383, 31034)	(5487, 21018)	(15343, 253)	
3245	(2679, 13152)	(2903, 20428)	(3733, 22670)	
2341, 3511	(189, 10738)	(245, 3885)	(315, 3843)	(441, 20440)
4683, 4685	(455, 12257)	(595, 23324)	(637, 21609)	(763, 27811)
11703, 14047	(875, 12971)	(1239, 26005)	(1449, 19292)	(1533, 27300)
	(1687, 3647)	(2919, 29869)	(3227, 23247)	(3367, 22841)
	(3493, 12754)	(3577, 16184)		
11627, order 5	(427, 25708)	(475, 3850)	(507, 10191)	(643, 5529)
	(663, 23704)	(867, 31974)	(959, 24578)	(1011, 7417)
	(1071, 23764)	(1445, 27753)	(1571, 31120)	(1609, 17207)
	(1967, 5767)	(1981, 11658)	(2259, 13289)	(2287, 10239)
	(2421, 8065)	(2443, 16062)	(2859, 5751)	(3485, 6137)
	(3743, 19913)	(5043, 14690)	(5439, 12401)	(7483, 19674)
	(2459, 13801)	(2797, 3116)	(3245, 7529)	(3375, 32292)
	(4013, 6976)	(6813, 27010)	(7023, 20199)	

Table 23 $n = 15$

$n = 15$ and $\alpha^{15} + \alpha + 1 = 0$				
t, ICNL	(τ, τ')			
14043	(145, 18628)	(269, 20897)	(307, 9489)	(409, 29773)
order 3	(437, 7387)	(447, 28347)	(451, 29528)	(619, 4300)
	(657, 21858)	(675, 20855)	(703, 5035)	(761, 21218)
	(775, 25166)	(811, 17819)	(825, 30447)	(859, 13028)
	(881, 19044)	(923, 6038)	(997, 3446)	(1063, 10189)
	(1083, 8016)	(1105, 25792)	(1133, 12359)	(1139, 21484)
	(1189, 30615)	(1207, 20211)	(1221, 13414)	(1301, 6136)
	(1321, 25537)	(1405, 27609)	(1419, 23864)	(1497, 18085)
	(1693, 16461)	(1739, 2627)	(1767, 8619)	(1977, 14723)
	(1991, 11496)	(2035, 14120)	(2201, 22465)	(2301, 23122)
	(2399, 9465)	(2463, 25358)	(2495, 24145)	(617, 2511)
	(2523, 16907)	(2533, 28396)	(2551, 23172)	(2607, 18776)
	(2645, 9384)	(2715, 25211)	(2733, 14359)	(1675, 2747)
	(2845, 8521)	(2889, 25600)	(2939, 19310)	(3027, 28506)
	(3065, 23146)	(3219, 4967)	(3223, 11223)	(3237, 9984)
	(3239, 3690)	(3247, 29159)	(3251, 17824)	(3295, 15492)
	(3303, 14144)	(3373, 30678)	(3475, 16165)	(3533, 15184)
	(3547, 4131)	(3559, 31005)	(3667, 16951)	(3671, 16459)
	(3737, 28515)	(179, 26858)	(3799, 13518)	(3961, 31539)
	(3965, 29171)	(4091, 23116)	(4693, 18461)	(4717, 16832)
	(5045, 8023)	(5095, 26408)	(5115, 31844)	(5311, 26741)
	(5325, 27322)	(5423, 20210)	(3187, 11434)	(5487, 21018)
	(921, 5501)	(5543, 29495)	(5837, 22201)	(5883, 26126)
	(5935, 22453)	(5939, 8381)	(5967, 17152)	(6639, 20337)
	(6653, 28576)	(207, 29525)	(4827, 21342)	(6975, 8080)
	(7003, 20554)	(1593, 19893)	(7159, 32621)	(7167, 10056)
	(7509, 11866)	(7605, 23046)	(7607, 26167)	(2517, 8175)
	(1201, 11003)	(11227, 22117)	(11639, 28176)	(3193, 25114)
	(3197, 7274)	(3321, 7275)	(3443, 19429)	(1621, 18108)
	(125, 20941)	(3877, 21053)	(4063, 11048)	(4077, 29570)
	(4755, 21437)	(4855, 23127)	(4943, 14208)	(5813, 15017)
	(6109, 17571)	(7485, 27499)	(7535, 32545)	(10971, 18539)

Table 24 $n = 16$

$n=16$ and $\alpha^{16}+\alpha^5+\alpha^3+\alpha^2+1=0$								
t, CNL								(τ, τ')
27	99,	141,	177,	309,	363,	453,	471	(11565*, 7967)
537,	609,	651,	687,	819,	873,	963,	1047	$*$, order 8
81119,	1161,	1197,	1383,	1491,	1629,	1671,	1707	
1839,	1893,	1983,	2139,	2181,	2217,	2349,	2403	
2493,	2511,	2649,	2727,	2859,	3003,	3021,	3159	
3423,	3513,	3531,	3669,	3711,	3747,	3879,	3933	
4023,	4041,	4389,	4533,	4551,	4731,	4899,	4953	
5043,	5061,	5277,	5463,	5787,	5919,	5973,	6063	
6429,	6483,	6573,	6591,	6729,	6771,	6807,	6939	
7083,	7101,	7167,	7317,	7503,	7593,	7611,	7677	
7827,	8013,	8103,	8121,	8187,	9651,	9717,	9831	
10143,	10227,	10653,	10671,	11181,	11247,	11583,	11691	
11757,	11871,	12093,	12267,	13623,	13947,	14079,	14967	
15099,	15327,	15837,	16119,	16347,	22239,	22383,	23259	
23487,	23931,	24507						
87,	93,	117,	171	213,	429,	441,	489	(2313*, 30840)
597,	603,	627,	681	723,	939,	951,	999	(28527*, 63222)
1107,	1113,	1137,	1191	1233,	1449,	1461,	1509	$*$, order 8
1617,	1623,	1647,	1701	1743,	1959,	1971,	2019	
2127,	2133,	2157,	2211	2253,	2469,	2481,	2637	
2643,	2667,	2763,	2979	2991,	3039,	3147,	3153	
3177,	3231,	3273,	3501	3549,	3657,	3663,	3687	
3741,	3783,	3999,	4011	4059,	4509,	4521,	4569	
4677,	4683,	4707,	4761	5019,	5031,	5079,	5271	
5529,	5541,	5589,	5703	5727,	5781,	5823,	6039	
6051,	6099,	6549,	6717	6747,	6843,	7059,	7071	
7311,	7353,	7581,	7629	7743,	7767,	7863,	8091	
8139,	9621,	9807,	9903	11199,	11709,	11901,	11943	
12159,	12219,	13179,	13239	13749,	13941,	13983,	14199	
15231,	15279,	15741,	15789	16239,	16251,	16299,	22359	
22527,	23391,	23547,	24063	24567,	28143,	30687		

Table 25 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, CNL	(τ, τ')			
11211	(927, 49915)	(2019, 58874)		
771	(4461, 21481)	(9693, 63620)	(23471, 8080)	
4527	(6969, 46322)	(7059, 58982)	(3403, 18022)	
1557	(405, 52658)	(11565, 7967)	(13695, 3814)	(4511, 27026)
11163	(3675, 52493)	(6735, 60203)	(11565, 7967)	(6521, 41126)
5397	(237, 62830) (14255, 58408)	(4863, 33268)	(3239, 44599)	(7787, 5170)
28527	(3273, 16328) (7549, 44270)	(13167, 12275)	(4435, 36464)	(6391, 11816)
10023	(2883, 398) (7525, 23474)	(12093, 37565) (8167, 20635)	(4969, 27824)	(5335, 62057)
2313	(3741, 42995) (7063, 65245)	(5679, 38249) (11477, 35585)	(6711, 9031)	(7353, 290)
4371 4377 12015 30591	(705, 2385) (5865, 31845)	(1845, 13575) (9675, 52050)	(2895, 42645) (11775, 54750)	(3915, 27060) (16125, 36870)

Table 26 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, CNL	(τ, τ')			
10923	(165, 64893)	(171, 31374)	(231, 18546)	(255, 12849)
	(285, 47526)	(291, 28329)	(321, 33171)	(339, 9984)
	(345, 15564)	(351, 35538)	(381, 32001)	(411, 18141)
	(417, 25671)	(423, 55596)	(453, 28257)	(465, 15702)
	(537, 41139)	(543, 567)	(627, 26052)	(645, 15594)
	(663, 31881)	(693, 61062)	(699, 40542)	(705, 2385)
	(723, 32691)	(741, 4506)	(789, 17994)	(837, 38058)
	(843, 52944)	(849, 38754)	(867, 10197)	(879, 60090)
	(885, 31809)	(933, 16683)	(999, 26601)	(1023, 59955)
	(1047, 4482)	(1095, 43242)	(1107, 4989)	(1113, 23346)
	(1125, 11199)	(1173, 13245)	(1191, 14874)	(1239, 40209)
	(1251, 31572)	(1269, 2682)	(1323, 50868)	(1329, 23463)
	(1371, 29115)	(1377, 51987)	(1395, 50802)	(1413, 64266)
	(1419, 12498)	(1497, 44001)	(1503, 35613)	(1509, 24429)
	(1515, 35283)	(1533, 2295)	(1551, 39075)	(1569, 53661)
	(1575, 60243)	(1581, 17745)	(1587, 51861)	(153, 1599)
	(1617, 41685)	(1641, 50268)	(1707, 14394)	(1713, 31698)
	(1731, 47937)	(1743, 14280)	(1755, 60021)	(1779, 39894)
	(1803, 36432)	(1845, 13575)	(1875, 25887)	(1881, 15909)
	(1899, 9423)	(1917, 3687)	(1941, 32214)	(2001, 62505)
	(2031, 49152)	(2037, 40011)	(2127, 49146)	(2139, 5565)
	(2163, 8637)	(2205, 22371)	(2313, 30840)	(2325, 37881)
	(2361, 24051)	(2379, 21918)	(2391, 4893)	(2427, 40524)
	(2535, 64338)	(2541, 41769)	(2583, 35004)	(2589, 48240)
	(2613, 44877)	(2643, 44196)	(2649, 50970)	(2679, 24060)
	(21, 8529)	(2703, 42684)	(2727, 4587)	(2757, 36402)
	(2775, 51057)	(2787, 3129)	(2805, 29544)	(2829, 58146)
	(2853, 44661)	(2859, 64098)	(2865, 24387)	(2895, 42645)
	(2919, 23613)	(2937, 55194)	(2967, 27129)	(2985, 30666)
	(2991, 45564)	(3021, 37464)	(3051, 62964)	(3159, 5739)
	(3195, 46584)	(3237, 31686)	(3261, 41697)	(3279, 4659)
	(3285, 62379)	(3291, 47613)	(3303, 23676)	(3321, 58260)
	(3387, 20115)	(3411, 26013)	(3429, 18831)	(3435, 64488)
	(3477, 57912)	(3483, 22899)	(3507, 48681)	(3645, 14001)
	(3723, 61929)	(3735, 61878)	(3765, 64794)	(3789, 47253)

Table 27 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, CNL	(τ, τ')			
Continue 10923	(3801, 7065)	(3909, 55908)	(3915, 27060)	(3963, 47994)
	(3987, 37647)	(3999, 18729)	(4041, 48756)	(4407, 56415)
	(4491, 59187)	(4521, 43281)	(4665, 32334)	(4719, 52224)
	(4725, 5421)	(4749, 44391)	(4845, 56778)	(4851, 18339)
	(4911, 40074)	(4947, 45384)	(4983, 13227)	(5013, 10038)
	(5037, 32205)	(5109, 36462)	(5283, 24249)	(5295, 58974)
	(5331, 44349)	(5427, 62199)	(5463, 23643)	(5481, 10137)
	(5535, 47499)	(5577, 60885)	(5595, 28677)	(5625, 39153)
	(5685, 53208)	(5721, 52143)	(5775, 12798)	(5835, 60027)
	(5865, 31845)	(5943, 25746)	(5967, 13806)	(5991, 53169)
	(6039, 10803)	(6063, 40338)	(6075, 12918)	(6381, 23274)
	(6387, 38223)	(6429, 43881)	(6447, 12057)	(6501, 7641)
	(6585, 53244)	(6615, 56154)	(6633, 13881)	(63, 36153)
	(6807, 47589)	(6837, 40398)	(6951, 49038)	(7083, 46371)
	(7119, 43002)	(7131, 59730)	(7341, 54249)	(7383, 51495)
	(7461, 51165)	(7629, 22350)	(7773, 56169)	(7845, 22419)
	(7869, 45012)	(7995, 40101)	(8043, 26286)	(8091, 24555)
	(8109, 20286)	(8121, 65532)	(8181, 19371)	(8187, 19035)
	(9429, 39348)	(9621, 27867)	(9645, 65196)	(9675, 52050)
	(10575, 59358)	(10707, 55275)	(10971, 57204)	(10995, 55014)
	(11067, 32103)	(11763, 59259)	(11775, 54750)	(11991, 43437)
	(13239, 28650)	(13533, 43734)	(13629, 52731)	(14139, 15159)
	(16125, 36870)	(28527, 63222)		

Table 28 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, ICNL	(τ, τ')			
2953	(10943, 40111)			
3181	(5587, 62678)	(5749, 33263)		
3925	(10231, 54272)	(10747, 38399)		
11231	(5515, 16004)	(7973, 20125)		
791 15677	(603, 61945)	(10221, 63356)		
931	(189, 53021)	(3069, 33593)	(3619, 6643)	
2827	(1869, 9310)	(5727, 41507)	(3805, 30487)	
3005	(6827, 45328)	(10919, 45868)	(13133, 56809)	
3749	(4557, 43730)	(6519, 33484)	(11605, 64102)	
5437	(3447, 21827)	(4581, 39055)	(10927, 60940)	
6389	(5403, 29246)	(10071, 6889)	(5861, 28172)	
6637	(5851, 42137)	(7037, 53833)	(11699, 47392)	
7657	(1671, 56720)	(7983, 15073)	(4985, 18353)	
8179	(4493, 59404)	(6131, 10624)	(12007, 64871)	
11623	(3177, 52813)	(11751, 51587)	(3487, 55576)	
13631	(4949, 17629)	(5593, 48755)	(10939, 25100)	
13727	(2661, 41192)	(13803, 1837)	(5983, 31849)	
23387	(927, 49915)	(2019, 58874)	(14847, 10922)	(3925, 54613)
1799	(2517, 53477)	(5499, 34915)	(4907, 1171)	(5759, 52558)
	(6029, 40055)	(16091, 934)		
4883	(909, 4459)	(1491, 46826)	(5757, 35800)	(7359, 30595)
	(3985, 42868)	(5449, 59470)		
7453	(681, 26684)	(4557, 43730)	(6519, 33484)	(21951, 25817)
	(3679, 45673)	(11605, 64102)		
11051	(483, 24217)	(2979, 36629)	(3711, 23342)	(5949, 56480)
	(3751, 51415)	(6733, 2869)		
13621	(2481, 33445)	(3219, 58229)	(10107, 31292)	(14031, 28205)
	(2951, 23165)	(3653, 38030)		

Table 29 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, ICNL	(τ, τ')			
3341	(1677, 20585)	(6741, 61775)	(11607, 47062)	(11871, 15241)
	(4571, 52055)	(10231, 54272)	(10747, 38399)	
7967	(501, 58715)	(1407, 45937)	(5859, 54130)	(7401, 36796)
	(3419, 29773)	(3481, 45707)	(6745, 57478)	(7321, 6820)
	(9839, 20266)			
9509	(633, 42511)	(3579, 11591)	(5301, 27830)	(11181, 1376)
	(6743, 31766)	(7883, 34174)	(9805, 64159)	(9811, 24224)
	(13723, 2471)			
12079	(957, 47669)	(1431, 23411)	(3057, 58073)	(8013, 35515)
	(3577, 45479)	(3731, 38027)	(4565, 18002)	(5207, 38419)
	(7541, 36919)			
15163	(261, 10667)	(6483, 53705)	(13671, 50101)	(24543, 50452)
	(3767, 50554)	(3803, 9157)	(4687, 20129)	(4751, 27740)
	(10859, 44267)			
2185	(705, 2385)	(1845, 13575)	(2895, 42645)	(3915, 27060)
3823	(5865, 31845)	(9675, 52050)	(11775, 54750)	(16125, 36870)
4373				
28399				
3277	(555, 24070)	(705, 2385)	(765, 16445)	(855, 30355)
13109	(1005, 34355)	(1065, 9565)	(1215, 37280)	(1695, 50710)
13111	(1725, 13100)	(1845, 13575)	(1965, 63730)	(1995, 48635)
13115	(2355, 35470)	(2655, 10430)	(2715, 22690)	(2745, 36115)
	(2895, 42645)	(2925, 23905)	(3045, 48305)	(3225, 5920)
	(3525, 60700)	(3795, 12545)	(3825, 35965)	(3915, 27060)
	(4815, 8930)	(4965, 55835)	(5865, 31845)	(6555, 11530)
	(7125, 27145)	(7545, 44015)	(8085, 36455)	(9405, 32020)
	(9675, 52050)	(9915, 44465)	(10035, 30875)	(10155, 14960)
	(10965, 24115)	(11775, 54750)	(13275, 28165)	(13935, 43120)
	(16125, 36870)	(16245, 55030)	(23535, 53380)	(3245, 61000)
	(3295, 39575)	(3385, 15565)	(3635, 45605)	(3685, 25870)
	(3785, 23855)	(3815, 42880)	(4045, 8485)	(4075, 60620)
	(4495, 35225)	(4535, 7780)	(265, 20105)	(4835, 8360)
	(5365, 28765)	(5695, 46315)	(5795, 5890)	(5845, 24310)
	(5975, 18410)	(5995, 10235)	(6125, 12880)	(6355, 38720)
	(7165, 44830)	(7355, 24425)	(7645, 60655)	(7795, 24430)
	(7835, 40810)	(8125, 62495)	(9445, 37015)	(545, 59300)
	(95, 59740)	(11495, 28030)	(155, 60925)	

Table 30 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, ICNL	(τ, τ')			
21847	(165, 64893)	(171, 31374)	(231, 18546)	(255, 12849)
	(285, 47526)	(291, 28329)	(321, 33171)	(339, 9984)
	(345, 15564)	(351, 35538)	(381, 32001)	(411, 18141)
	(417, 25671)	(423, 55596)	(453, 28257)	(465, 15702)
	(537, 41139)	(543, 567)	(627, 26052)	(645, 15594)
	(663, 31881)	(693, 61062)	(699, 40542)	(705, 2385)
	(723, 32691)	(741, 4506)	(789, 17994)	(837, 38058)
	(843, 52944)	(849, 38754)	(867, 10197)	(879, 60090)
	(885, 31809)	(933, 16683)	(999, 26601)	(1023, 59955)
	(1047, 4482)	(1095, 43242)	(1107, 4989)	(1113, 23346)
	(1125, 11199)	(1173, 13245)	(1191, 14874)	(1239, 40209)
	(1251, 31572)	(1269, 2682)	(1323, 50868)	(1329, 23463)
	(1371, 29115)	(1377, 51987)	(1395, 50802)	(1413, 64266)
	(1419, 12498)	(1497, 44001)	(1503, 35613)	(1509, 24429)
	(1515, 35283)	(1533, 2295)	(1551, 39075)	(1569, 53661)
	(1575, 60243)	(1581, 17745)	(1587, 51861)	(153, 1599)
	(1617, 41685)	(1641, 50268)	(1707, 14394)	(1713, 31698)
	(1731, 47937)	(1743, 14280)	(1755, 60021)	(1779, 39894)
	(1803, 36432)	(1845, 13575)	(1875, 25887)	(1881, 15909)
	(1899, 9423)	(1917, 3687)	(1941, 32214)	(2001, 62505)
	(2031, 49152)	(2037, 40011)	(2127, 49146)	(2139, 5565)
	(2163, 8637)	(2205, 22371)	(2325, 37881)	(2361, 24051)
	(2379, 21918)	(2391, 4893)	(2427, 40524)	(2535, 64338)
	(2541, 41769)	(2583, 35004)	(2589, 48240)	(2613, 44877)
	(2643, 44196)	(2649, 50970)	(2679, 24060)	(21, 8529)
	(2703, 42684)	(2727, 4587)	(2757, 36402)	(2775, 51057)
	(2787, 3129)	(2805, 29544)	(2829, 58146)	(2853, 44661)
	(2859, 64098)	(2865, 24387)	(2895, 42645)	(2919, 23613)
	(2937, 55194)	(2967, 27129)	(2985, 30666)	(2991, 45564)
	(3021, 37464)	(3051, 62964)	(3159, 5739)	(3195, 46584)
	(3237, 31686)	(3261, 41697)	(3279, 4659)	(3285, 62379)
	(3291, 47613)	(3303, 23676)	(3321, 58260)	(3387, 20115)
	(3411, 26013)	(3429, 18831)	(3435, 64488)	(3477, 57912)
	(3483, 22899)	(3507, 48681)	(3645, 14001)	(3723, 61929)
	(3735, 61878)	(3765, 64794)	(3789, 47253)	(3801, 7065)
	(3909, 55908)	(3915, 27060)	(3963, 47994)	(3987, 37647)
	(3999, 18729)	(4041, 48756)	(4407, 56415)	(4491, 59187)

Table 31 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, ICNL	(τ, τ')			
Continue	(4521, 43281)	(4665, 32334)	(4719, 52224)	(4725, 5421)
21847	(4749, 44391)	(4845, 56778)	(4851, 18339)	(4911, 40074)
	(4947, 45384)	(4983, 13227)	(5013, 10038)	(5037, 32205)
	(5109, 36462)	(5283, 24249)	(5295, 58974)	(5331, 44349)
	(5427, 62199)	(5463, 23643)	(5481, 10137)	(5535, 47499)
	(5577, 60885)	(5595, 28677)	(5625, 39153)	(5685, 53208)
	(5721, 52143)	(5775, 12798)	(5835, 60027)	(5865, 31845)
	(5943, 25746)	(5967, 13806)	(5991, 53169)	(6039, 10803)
	(6063, 40338)	(6075, 12918)	(6381, 23274)	(6387, 38223)
	(6429, 43881)	(6447, 12057)	(6501, 7641)	(6585, 53244)
	(6615, 56154)	(6633, 13881)	(63, 36153)	(6807, 47589)
	(6837, 40398)	(6951, 49038)	(7083, 46371)	(7119, 43002)
	(7131, 59730)	(7341, 54249)	(7383, 51495)	(7461, 51165)
	(7629, 22350)	(7773, 56169)	(7845, 22419)	(7869, 45012)
	(7995, 40101)	(8043, 26286)	(8091, 24555)	(8109, 20286)
	(8121, 65532)	(8181, 19371)	(8187, 19035)	(9429, 39348)
	(9621, 27867)	(9645, 65196)	(9675, 52050)	(10575, 59358)
	(10707, 55275)	(10971, 57204)	(10995, 55014)	(11067, 32103)
	(11763, 59259)	(11775, 54750)	(11991, 43437)	(13239, 28650)
	(13533, 43734)	(13629, 52731)	(14139, 15159)	(16125, 36870)

Table 32 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, ICNL	(τ, τ')			
4369	(177, 64496)	(279, 33067)	(309, 60787)	(363, 58409)
order 4	(441, 54463)	(747, 5291)	(981, 11668)	(1119, 10942)
	(1263, 51509)	(1311, 11593)	(1623, 19589)	(1833, 38309)
	(2007, 20801)	(2013, 63794)	(2121, 54343)	(2169, 56767)
	(2187, 24551)	(2223, 2369)	(2247, 26546)	(2331, 54523)
	(2673, 23504)	(2751, 28618)	(2961, 37813)	(2997, 39596)
	(3171, 31213)	(3567, 36266)	(3663, 11894)	(3867, 29186)
	(3957, 8201)	(4017, 27716)	(4563, 50879)	(4653, 41264)
	(4797, 48611)	(5277, 62726)	(5373, 49169)	(5751, 30298)
	(5979, 20227)	(6747, 56861)	(6759, 64837)	(6879, 53137)
	(7017, 22931)	(1169, 36318)	(1747, 18174)	(7347, 56411)
	(7659, 48802)	(7911, 27613)	(7989, 51397)	(8157, 36866)
	(9399, 61777)	(9909, 22927)	(2831, 37782)	(1631, 10167)
	(10239, 18352)	(10587, 23366)	(10749, 32197)	(11061, 28528)
	(11091, 60688)	(11259, 64432)	(11997, 14576)	(14799, 55462)
	(15357, 65516)	(5591, 15837)	(12011, 24567)	(28143, 64919)
	(28671, 53098)	(3049, 43051)	(3121, 5329)	(3245, 61000)
	(3295, 39575)	(3563, 36512)	(3815, 42880)	(3857, 57833)
	(4039, 60661)	(4043, 34172)	(4075, 60620)	(4495, 35225)
	(4535, 7780)	(27, 63011)	(4763, 63197)	(265, 20105)
	(4847, 15938)	(4939, 16441)	(5027, 45503)	(197, 17693)
	(5795, 5890)	(5995, 10235)	(6125, 12880)	(6355, 38720)
	(7013, 40412)	(7069, 43831)	(7835, 40810)	(8117, 57338)
	(8125, 62495)	(8183, 43637)	(2371, 15049)	(137, 58538)
	(173, 58547)	(9677, 28838)	(9917, 25613)	(10651, 28789)
	(10907, 34883)	(11117, 32093)	(95, 59740)	(11495, 28030)
	(11573, 49592)	(15311, 32802)	(9, 63157)	(24443, 46847)
	(155, 60925)			

Table 33 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, ICNL	(τ, τ')			
30583	(207, 41986)	(297, 17641)	(303, 26104)	(333, 30739)
order 4	(459, 50057)	(609, 41462)	(651, 51368)	(861, 19403)
	(969, 34262)	(1179, 52847)	(1629, 48707)	(1683, 34189)
	(1701, 48683)	(1719, 4772)	(1749, 37622)	(1947, 18391)
	(2403, 26434)	(2487, 4816)	(2763, 6679)	(2877, 28546)
	(3027, 5521)	(3189, 63212)	(3207, 13156)	(241, 57747)
	(3381, 45428)	(3519, 33632)	(3771, 23993)	(3813, 55144)
	(4077, 38476)	(4083, 14899)	(4371, 23228)	(4527, 34876)
	(4773, 10444)	(5061, 63803)	(5271, 22853)	(1277, 35514)
	(2393, 35526)	(5529, 8912)	(2537, 5589)	(5607, 61396)
	(5787, 13741)	(5811, 50303)	(6093, 13759)	(6117, 29251)
	(6783, 57184)	(6969, 46322)	(7059, 58982)	(7533, 51349)
	(7791, 42203)	(7881, 8483)	(7977, 41341)	(9567, 33841)
	(10059, 37112)	(10203, 42574)	(10653, 47509)	(541, 51882)
	(11103, 61654)	(11211, 65168)	(11679, 57167)	(11943, 20434)
	(12267, 26476)	(13173, 39814)	(13557, 19471)	(13653, 14194)
	(14829, 48827)	(15291, 21098)	(11071, 15867)	(16221, 29063)
	(22203, 35269)	(271, 23547)	(2939, 62981)	(3245, 61000)
	(3295, 39575)	(3373, 58477)	(3403, 18022)	(3551, 51749)
	(3733, 23257)	(3815, 42880)	(3839, 41696)	(3881, 14984)
	(4057, 4723)	(4075, 60620)	(4079, 50606)	(4495, 35225)
	(4535, 7780)	(4537, 48223)	(4559, 47546)	(4661, 34829)
	(265, 20105)	(5309, 63806)	(5531, 51224)	(5789, 32951)
	(5795, 5890)	(5833, 56932)	(5939, 22271)	(5995, 10235)
	(6067, 17428)	(6125, 12880)	(6355, 38720)	(6569, 49181)
	(7093, 43627)	(7357, 32278)	(7483, 31462)	(7567, 36943)
	(7769, 64421)	(7835, 40810)	(8125, 62495)	(10151, 22544)
	(10571, 33719)	(95, 59740)	(11495, 28030)	(12269, 59126)
	(973, 39757)	(2009, 48041)	(14261, 27659)	(14831, 46724)
	(1213, 22267)	(155, 60925)		

Table 34 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, ICNL	(τ, τ')			
21845	(2923, 3350)	(2935, 18668)	(2941, 60782)	(2959, 3773)
order 2	(2963, 12292)	(2983, 35555)	(2995, 31448)	(3005, 40744)
	(3013, 18419)	(3017, 7141)	(3019, 14000)	(3025, 44621)
	(3047, 61834)	(3061, 39215)	(3067, 7265)	(3131, 18652)
	(3143, 9433)	(3145, 31196)	(3155, 14113)	(3157, 49703)
	(3173, 20128)	(3199, 46805)	(3209, 41137)	(3223, 55544)
	(289, 49958)	(3229, 42710)	(3239, 44599)	(3245, 61000)
	(263, 34408)	(3295, 39575)	(3299, 61030)	(3305, 5887)
	(575, 3313)	(3317, 54898)	(3319, 61598)	(3325, 64271)
	(3343, 49751)	(3349, 34031)	(3371, 36916)	(3377, 61894)
	(3389, 47434)	(3391, 60650)	(3401, 64870)	(3419, 29773)
	(3421, 19028)	(3445, 63218)	(3449, 16183)	(3451, 56462)
	(3475, 32987)	(3481, 45707)	(3491, 20398)	(3493, 11885)
	(3497, 21112)	(3509, 34588)	(473, 34525)	(449, 17263)
	(3527, 8707)	(3535, 54467)	(3547, 58340)	(3557, 31843)
	(3577, 45479)	(3629, 39535)	(1699, 34586)	(3647, 25891)
	(3661, 52505)	(3667, 63542)	(3671, 21967)	(3673, 27008)
	(3683, 18331)	(3689, 40525)	(3695, 15076)	(3701, 6748)
	(3721, 4403)	(3725, 7351)	(3737, 48436)	(3743, 46714)
	(3761, 27040)	(3763, 59732)	(3767, 50554)	(3775, 47447)
	(3781, 14189)	(3787, 30395)	(3797, 26506)	(3803, 9157)
	(3815, 42880)	(3827, 19957)	(3865, 21191)	(3875, 19567)
	(3887, 35668)	(3911, 57394)	(3917, 19582)	(3941, 64708)
	(3953, 24916)	(3955, 29351)	(3959, 31669)	(3967, 27533)
	(3979, 32348)	(3983, 42007)	(3995, 8764)	(4015, 62327)
	(4025, 46738)	(2027, 34795)	(4063, 23594)	(4075, 60620)
	(4081, 44828)	(4373, 55279)	(4393, 57824)	(4411, 47501)
	(4415, 64873)	(4421, 14134)	(4423, 53780)	(4435, 36464)
	(4477, 58058)	(4493, 59404)	(4495, 35225)	(4505, 7804)
	(4519, 36194)	(4523, 29593)	(4531, 27428)	(4535, 7780)
	(4585, 60026)	(79, 53534)	(4591, 32150)	(4645, 15968)
	(4649, 5569)	(4679, 53062)	(4681, 31226)	(4685, 28198)
	(4687, 20129)	(4697, 60154)	(4699, 25937)	(4703, 44779)
	(4709, 4822)	(4711, 45047)	(1259, 4729)	(4747, 12365)
	(265, 20105)	(4781, 30022)	(4783, 35669)	(4789, 14387)
	(4799, 30373)	(527, 50356)	(4823, 65476)	(4841, 6712)

Table 35 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, ICNL	(τ, τ')			
Continue	(4843, 59414)	(4855, 32453)	(4885, 8276)	(1171, 4907)
21845	(4925, 51547)	(4937, 36841)	(4943, 32518)	(4949, 17629)
	(4969, 27824)	(4973, 49330)	(4987, 32834)	(4991, 33346)
	(5021, 42202)	(5039, 35458)	(5047, 35522)	(5051, 19105)
	(5065, 56279)	(5071, 34034)	(5075, 12043)	(331, 35306)
	(5081, 50734)	(5083, 57344)	(5099, 33850)	(5207, 38419)
	(5209, 53426)	(5221, 23765)	(5231, 20011)	(5269, 22154)
	(5275, 62159)	(5323, 27287)	(5335, 62057)	(5341, 59438)
	(5357, 11221)	(5359, 44789)	(5371, 36365)	(5453, 64279)
	(5459, 7720)	(5479, 31190)	(5491, 15053)	(5501, 61375)
	(5515, 16004)	(5527, 52964)	(5539, 6389)	(5545, 20537)
	(5561, 15472)	(5567, 48922)	(5579, 16795)	(5587, 62678)
	(5593, 48755)	(5605, 64466)	(5615, 49726)	(5693, 53023)
	(5743, 59834)	(5749, 33263)	(5755, 28379)	(5759, 52558)
	(5795, 5890)	(5797, 60446)	(5813, 19378)	(5851, 42137)
	(5869, 20231)	(1291, 31115)	(5935, 58016)	(5977, 36506)
	(5995, 10235)	(5999, 10912)	(6037, 47591)	(6043, 52433)
	(6055, 23027)	(6059, 16807)	(6071, 64621)	(6091, 31424)
	(6119, 60733)	(6121, 10964)	(6125, 12880)	(6131, 10624)
	(6349, 15395)	(6355, 38720)	(6391, 11816)	(6445, 62333)
	(6451, 33065)	(6455, 26989)	(6457, 16859)	(6485, 48253)
	(6487, 49238)	(6511, 65096)	(6515, 41113)	(6551, 42340)
	(6601, 24245)	(6607, 37763)	(6637, 36863)	(6655, 36794)
	(157, 41798)	(6719, 10567)	(6727, 36527)	(1133, 6739)
	(6767, 45685)	(6773, 59413)	(6799, 30596)	(6809, 53884)
	(6821, 53407)	(6827, 45328)	(2899, 6839)	(6859, 53303)
	(6863, 40510)	(6869, 20425)	(6949, 55349)	(6959, 60619)
	(457, 36260)	(7001, 9211)	(7019, 48043)	(7037, 53833)
	(7081, 56474)	(7129, 42554)	(7285, 50048)	(7289, 7690)
	(7327, 31385)	(7337, 32275)	(7381, 62687)	(7385, 51718)
	(7411, 33761)	(7415, 31951)	(7471, 50216)	(7487, 38983)
	(7501, 46889)	(7525, 23474)	(7541, 36919)	(7549, 44270)
	(7577, 17002)	(7589, 48541)	(2521, 36569)	(7607, 63391)
	(7615, 14123)	(7627, 28535)	(7639, 47129)	(7679, 21466)

Table 36 $n = 16$

$n = 16$ and $\alpha^{16} + \alpha^5 + \alpha^3 + \alpha^2 + 1 = 0$				
t, ICNL	(τ, τ')			
Continue	(2585, 36661)	(7805, 8596)	(7829, 48469)	(7835, 40810)
21845	(7853, 40357)	(7883, 34174)	(7925, 14077)	(7933, 40556)
	(7973, 20125)	(7981, 51104)	(7987, 27143)	(7991, 18484)
	(8063, 55072)	(8089, 38645)	(8093, 31693)	(1607, 18409)
	(8125, 62495)	(8159, 12460)	(8173, 8537)	(8189, 64552)
	(9365, 43732)	(517, 9383)	(9391, 38519)	(9419, 11689)
	(9517, 37727)	(9527, 17680)	(9533, 27328)	(9557, 16984)
	(9563, 49774)	(1049, 24139)	(9655, 16967)	(65, 62614)
	(1441, 11897)	(9719, 52267)	(9811, 24224)	(9839, 20266)
	(9877, 36959)	(9947, 28135)	(9979, 62681)	(10093, 45719)
	(89, 43327)	(10231, 54272)	(10237, 19901)	(10589, 40870)
	(10663, 43877)	(10711, 17480)	(511, 12959)	(10741, 23168)
	(10747, 38399)	(10751, 43513)	(10879, 29945)	(10919, 45868)
	(10939, 25100)	(10943, 40111)	(10967, 45013)	(10973, 63136)
	(11069, 55555)	(11093, 49273)	(11101, 44858)	(479, 11119)
	(11125, 18461)	(11131, 23990)	(11135, 49849)	(11159, 52138)
	(95, 59740)	(239, 62137)	(11261, 31654)	(11455, 18977)
	(11495, 28030)	(11599, 12440)	(11699, 47392)	(997, 46949)
	(11711, 55795)	(11893, 43265)	(11951, 32167)	(11957, 49495)
	(11965, 15989)	(727, 52151)	(12007, 64871)	(12019, 15605)
	(12029, 29104)	(12157, 27668)	(12199, 46571)	(13133, 56809)
	(13211, 55711)	(13243, 22517)	(13531, 56840)	(13661, 23059)
	(2471, 13723)	(13783, 57488)	(13787, 57856)	(13799, 30340)
	(14191, 17033)	(14203, 25619)	(14255, 58408)	(15019, 16949)
	(15197, 34672)	(15319, 22790)	(15323, 51808)	(15359, 16447)
	(1643, 65146)	(233, 59893)	(467, 40813)	(16093, 35591)
	(16235, 52228)	(16319, 18454)	(16381, 27392)	(21851, 41059)
	(22363, 43535)	(2243, 43957)	(22463, 33253)	(2479, 54782)
	(505, 62906)	(23479, 24590)	(311, 55165)	(737, 28543)
	(155, 60925)	(30719, 63511)	(655, 65519)	

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